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# COMPUTATIONAL EFFICIENCY OF SEARCHING KEMENY CONSENSUS AND ENROLLMENT OF EXPERTS FOR PREFERRING ON STOCK-MARKETS

***It is studied where the algorithm's MATLAB code for searching Kemeny ranking/consensus is effective. Besides, a matter of minimizing the number of experts is discussed. Statistics of searching Kemeny ranking/consensus are obtained in MATLAB. The algorithm's effectiveness becomes more apparent for greater number of objects. Its MATLAB code is very simple, so the effectiveness is expected when other programming environments will be used (say, C/C++, Python, Java). A matter of minimizing the number of experts is reduced to using the simplest voting via social networks. Such social networks can be organized as corporative groups/communities in Facebook or other known networks. The stated result can be used in making preferences on options or other related stock-market objects.***

***Keywords: preferences, Kemeny ranking/consensus, algorithm's effectiveness, enrollment of experts.***

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# ОБЧИСЛЮВАЛЬНА ЕФЕКТИВНІСТЬ ПОШУКУ КОНСЕНСУСУ КЕМЕНІ І НАБІР ЕКСПЕРТІВ ДЛЯ ВИСУНЕННЯ ПЕРЕВАГ НА ФОНДОВИХ РИНКАХ

***Вивчається те, де MATLAB-код алгоритму для пошуку ранжування/консенсусу Кемені є ефективним. Окрім цього, обговорюється питання мінімізації числа експертів. Статистика пошуку ранжування/консенсусу Кемені отримується в MATLAB. Ефективність алгоритму стає більш очевидною для більшого числа об'єктів. Його MATLAB-код дуже простий, тому очікується така сама ефективність в інших середовищах (скажімо, C/C++, Python, Java). Питання мінімізації числа експертів зводиться до використання найпростішого голосування у соціальних мережах. Такі соціальні мережі можуть бути організовані як корпоративні групи/товариства у Facebook або інших відомих мережах. Викладений результат може бути застосований у виробленні переваг на опціонах або інших об'єктах фондових ринків.***

***Ключові слова: переваги, ранжування/консенсус Кемені, ефективність алгоритму, набір експертів.***

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# ВЫЧИСЛИТЕЛЬНАЯ ЭФФЕКТИВНОСТЬ ПОИСКА КОНСЕНСУСА КЕМЕНИ И НАБОР ЭКСПЕРТОВ ДЛЯ ВЫДВИЖЕНИЯ ПРЕДПОЧТЕНИЙ НА ФОНДОВЫХ РЫНКАХ

***Изучается то, где MATLAB-код алгоритма для поиска ранжирования/консенсуса Кемени является эффективным. Кроме этого, обсуждается вопрос минимизации числа экспертов. Статистика поиска ранжирования/консенсуса Кемени получена в MATLAB. Эффективность алгоритма стаёт более очевидной для большего числа объектов. Его MATLAB-код очень простой, поэтому ожидается та же эффективность в других средах (скажем, C/C++, Python, Java). Вопрос минимизации числа экспертов сводится к использованию простейшего голосования в социальных сетях. Такие социальные сети могут быть организованы как корпоративные группы/товарищества в Facebook или других известных сетях. Изложенный результат может быть использован при вырабатывании предпочтений на опционах или других объектах фондовых рынков.***

***Ключевые слова: предпочтения, ранжирование/консенсус Кемени, эффективность алгоритма, набор экспертов.***

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**Introduction.** A great number of economical processes deal with selecting preferences and decision-making. Preferences are needed to form sequences of options/features/criteria and relation among them, whereupon decisions are made. Surely, experts are invoked into those processes of voting and preferring. For aggregating the experts' judgments, a Kemeny consensus is searched. This is not about simple averaging, as experts' judgments are rarely represented as single-valued points. In practice, the judgments are results of pairwise comparisons. Then, square matrices are formed, where every expert's judgment corresponds to one's matrix. Thus, a Kemeny consensus is searched over square matrices. However, if the consensus is found, it should be an acyclic ranking, i. e. a matrix corresponding to the Kemeny consensus must be converted back into a definite sequence. A complex question is that a Kemeny consensus be a single, and be searched fast, with invoking not many experts.

**Analysis of the previous works.** A problem of ranking a finite set of objects into a Kemeny ranking/consensus is NP-hard [1, 2]. For cases with matrices, a consensus matrix is searched over a finite set of permutations each of which shows a ranking/matrix, and this consensus matrix should be as close as possible to a set of experts' rankings/matrices. In the article [3], an algorithm that speeds up the search of the Kemeny consensus is represented. For that, an approach for aggregating experts' rankings is suggested and substantiated. Also a simple metric to compare rankings is suggested and substantiated. The developed algorithm finds a set of Kemeny rankings much faster than the classical straightforward search. Also this set often contains a single Kemeny consensus, unlike the straightforward search giving more than one consensus. Besides, a single Kemeny consensus is determined at one stroke if the averaged expert ranking turns out acyclic. Thus, the article [3] solves the problem of selecting a single Kemeny consensus. However, a question of the speed gain along with minimizing a number of experts (to save both computational and financial resources) remains open.

**Goal of the article.** The goal is to ascertain whether it is possible to minimize a number of experts invoked for voting/judging. At that, a solution by the method in the article [3] can coincide with the solution by the straightforward search. Selection of an effective program code is welcomed also. For achieving the goal, we have to fulfill the following tasks:

1. To gather statistics of searching Kemeny ranking/consensus.

2. To decide on when a number of experts can be minimized (issuing also from running effectively a program code routine of searching Kemeny ranking/consensus).

**Statistics of searching Kemeny ranking/consensus.** Suppose that we consider $N$ economical objects to be ranked. Let $\mathbf{E}_k(N) = \left[ m_{ij}^{\langle k \rangle} \right]_{N \times N}$ be a matrix, which is formed from a sequence given by the $k$-th expert, $k = \overline{1, K}$ ($K$ is a total number of experts). We know that

$$m_{jj}^{\langle k \rangle} = 0 \quad \forall\ j = \overline{1, N} \ \text{ and } \ \forall\ k = \overline{1, K} \ \text{ by } \ m_{ij}^{\langle k \rangle} = \pm 1 \ \text{ and } \ m_{ij}^{\langle k \rangle} = -m_{ji}^{\langle k \rangle}, \tag{1}$$

i. e. $\mathbf{E}_k(N) = -\left(\mathbf{E}_k(N)\right)^{\mathrm{T}}$. A Kemeny ranking/consensus is searched as

$$\tilde{\mathbf{E}}(N) \in \arg \min_{\substack{\mathbf{A}_a(N) \in A_N(\pm 1) \\ a = 1, \left|A_N(\pm 1)\right|}} \left\{ \sum_{k=1}^{K} \varphi_k \cdot \rho_{A_N(\pm 1)}\left(\mathbf{E}_k(N), \mathbf{A}_a(N)\right) \right\}, \tag{2}$$

where the following denotations are used:

* $A_N(\pm 1)$ is a space of all square matrices whose order is $N$ and its elements satisfy conditions (1);

* $\varphi_k$ is a weight of the $k$-th expert [4], corresponding to the proficiency of this expert, and $\varphi_k \in (0; 1)$ by $\sum_{k=1}^{K} \varphi_k = 1$;

* $\rho_{A_N(\pm 1)}$ is a metric/distance between two matrices from the space $A_N(\pm 1)$ (e. g., see peculiarities relating to such distance in [1, 3, 4, 6]).

A case with $N = 3$ is primitive, but even this gives us a concern. Figure 1, where normalized time of the method in the article [3] and its time gain are plotted versus a number of experts, shows that a tender tendency of their growth exists. Number of experts varies from 5 to 205 with a step of 2. Note that the gain is not always greater than 1. Moreover, when we have less than 50 experts, the gain is mostly less than 1, which is a negative effect. Increasing the number of experts does not change the tendency much. Therefore, it is sufficient to invoke 100 to 160 experts for ranking three objects.

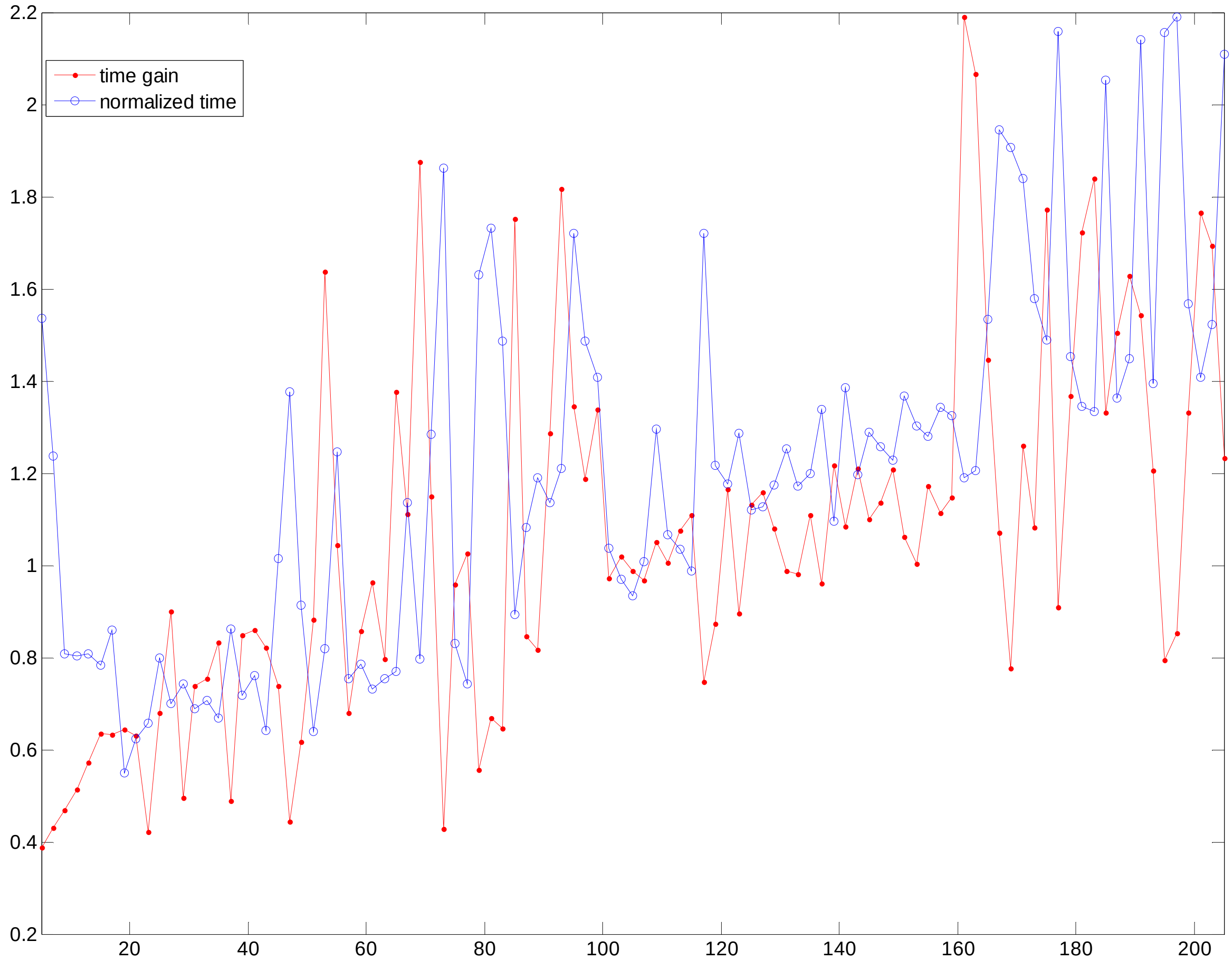


**Figure 1. Growing gain and time for searching the Kemeny consensus for three objects (a program code routine of searching Kemeny ranking/consensus is run in MATLAB)**

For a case with $N = 4$, when we invoke 5 to 105 experts, the tendency remains similar (Figure 2). Nevertheless, here the gain is not so scattered. For ranking four objects, it is sufficient to invoke 70 to 100 experts, that is much less than in the case with ranking three objects. By the way, the time now is more stable, and it does not grow much (unlike that in Figure 1).

It should be noticed that sometimes the algorithm from [3] fails (see those peaks-and-drops in Figure 3), although time taken for computations still increases not much. Another general conclusion is that the time increases too slowly (if to exclude those fails with peaks-and-drops), that implies the algorithm is effective. Its MATLAB code is very simple, so the effectiveness is expected when other programming environments will be used (say, C/C++, Python, Java). For greater number of objects to be ranked, this tendency becomes better (Figure 4) — the time slightly even decreases, while the gain increases constantly.

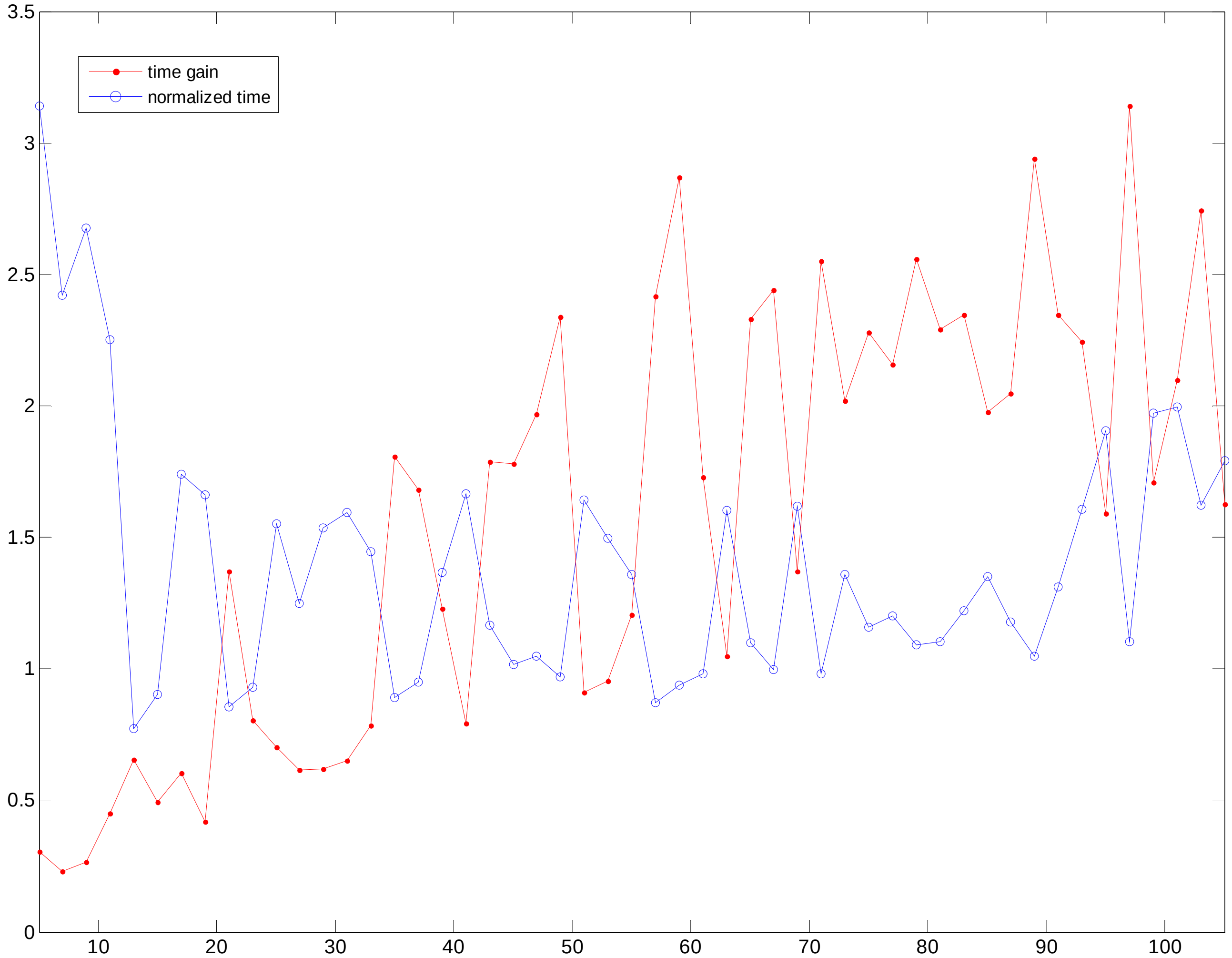


**Figure 2. More stable time of computations in searching Kemeny ranking/consensus in MATLAB for ranking four objects**

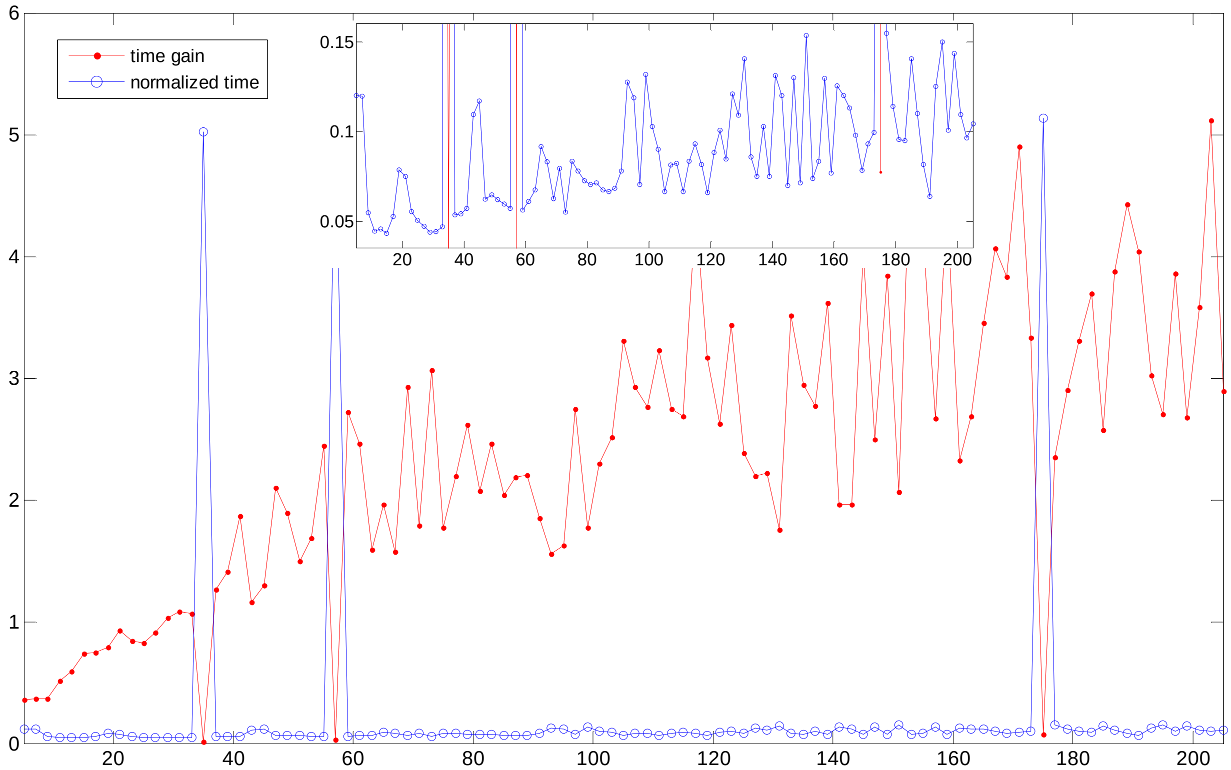


**Figure 3. Unexpected peaks of time and the corresponding drops of gain ( $N = 4$ )**

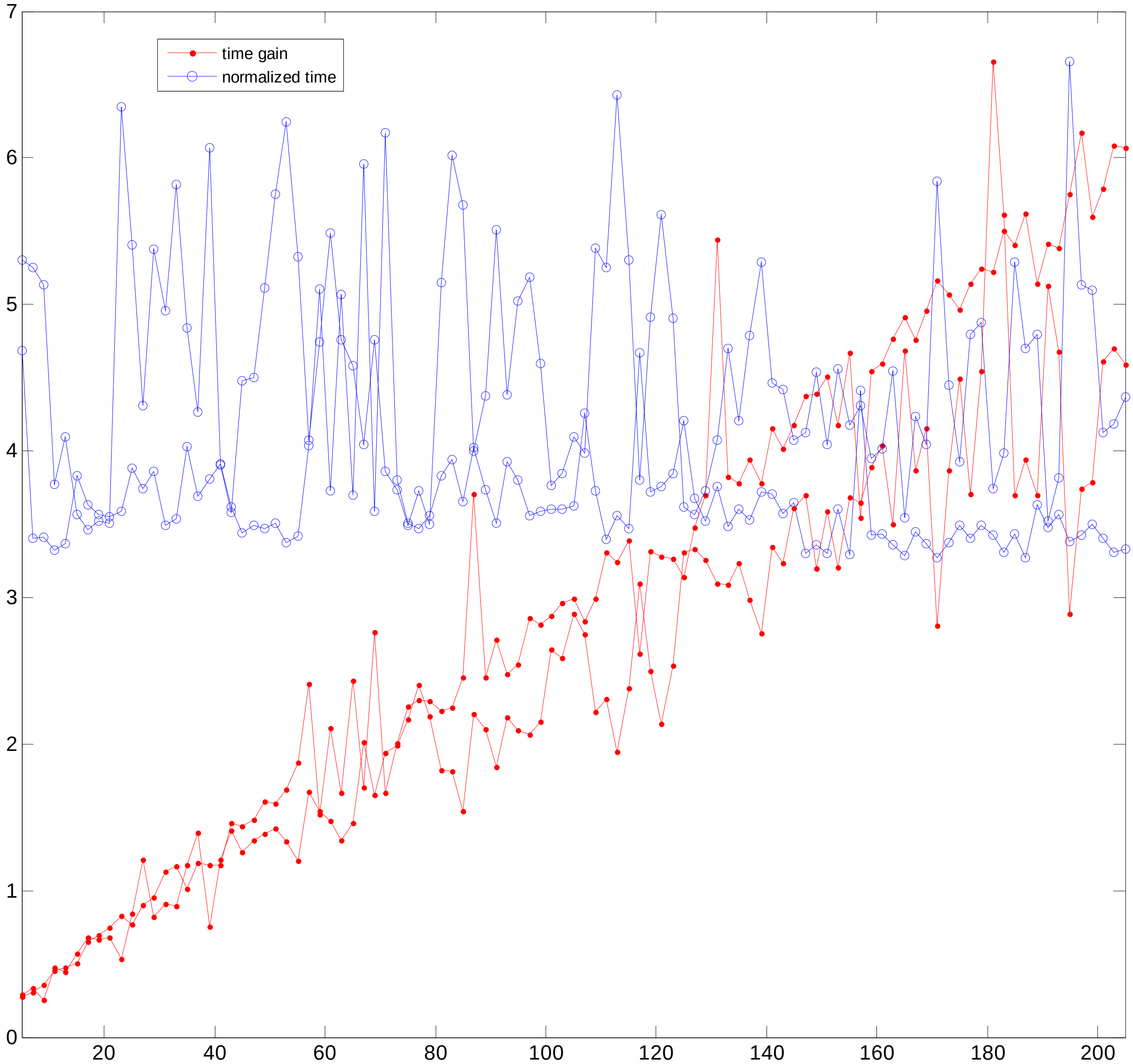


**Figure 4. Growth of the gain and the corresponding time for $N = 5$ and $N = 6$**

Is there any opportunity to minimize number of experts based on those results? Obviously, it depends on additional constraints.

**A matter of minimizing the number of experts.** For now, we know that the algorithm's MATLAB code for searching Kemeny ranking/consensus is effective. Its effectiveness becomes more apparent for greater number of objects. To decide on a number of experts, we must learn whether any connection between the gain-and-the-corresponding-time of the algorithm and the solution's accuracy exists. However, such connection is unclear. Instead of that, we may use a technique suggested in [7]. That approach suggests that experts give their judgments simply in binary form, wherein the expert's judgment in comparing two objects/entities is either 1 or 0. This can be easily implemented via social networks with setting "like"/"dislike" (advantage is "like" or 1, disadvantage is "dislike" or 0). Moreover, "experts" actually should not be necessarily so proficient. The eventual result (the average ranking over experts' rankings) is truly expected to be close to reality owing to the law of large numbers. Therefore, we rather need to invoke more experts using social networks, that will give us finally well enough judgments.

**Conclusion.** Based on the statistics of searching Kemeny ranking/consensus, the algorithm's MATLAB code efficiency has been shown. The efficiency depends on the number of experts. This dependence is stronger than that of the number of objects to be ranked. A matter of minimizing the number of experts is reduced to using the simplest voting via social networks. Such social networks can be organized as corporative groups/communities in Facebook or other known networks. So, the only task is to find a Kemeny consensus, where avoiding the classical straightforward search by (2) becomes then crucial (as number of a network users/“experts” will be large). The stated result can be used in making preferences on options or other related stock-market objects.

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