INVERSE PROBLEMS FOR ENSURING THE QUALITY OF COMPLEX TECHNICAL SYSTEMS USING MULTI-CRITERIA OPTIMIZATION

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Creation of any complex technical system, for example, a new machine, mechanism, technological, medical-biological and other systems and processes begins with setting technical specifications (TS) for output parameters. These conditions are expressed in the form of nominal values of output parameters and tolerances on their values [1–3].

Let the quality of any object be evaluated by the values of its output parameters $\mathbf{y} = \left(y_1, y_2, ..., y_m\right)^T$. To ensure a given level of quality means to guarantee the fulfillment of the following relations:

$$\left[y_i\right] \le y_i \le \left[Y_i\right],\ i = 1, 2, ..., m\ , \tag{1}$$

where $\left[y_i\right]$, $\left[Y_i\right]$ are the lower and upper limits of the parameter , specified in specifications.

The solution of this problem should be sought in the form of a set of values of primary factors, which can be represented by the inequality:

$$\left[x_i\right] \le x_i \le \left[X_i\right],\ i = 1, 2, ..., n\ , \tag{2}$$

determining the area of system operability.

Belonging to this set should ensure the fulfillment of constraints (1) imposed on the output parameters of the object. The formulated problem will be further referred to as a multiple inverse problem, thus emphasizing that its solution implies the definition of a set of values (area) in the space of primary factors.

A set of methods for solving the problem

Reducing to an optimization problem. The formation of the vector y is completely determined by the set of the vector of values of primary factors $\mathbf{x} = \left(x_1, x_2, ..., x_n\right)^T$ using the operator:

$$\mathbf{y} = \mathbf{f}\left(\mathbf{x}, \mathbf{B}\right), \tag{3}$$

making the connection between these vectors.

The structure $\mathbf{f}=\left(f_1,f_2,...,f_m\right)$ and vector of parameters of the mathematical model $\mathbf{B}=\left(B_1,B_2,...,B_k\right)$ correspond to the physical nature and functional purpose of the object.

As a rule, production, physical, economic and other considerations allow us to specify wide limits of sets of possible values of primary factors. Then the system (1) supplemented by these constraints, taking into account the coordinate form of the operator (3), can be written in the following form:

$$\begin{cases}\mathbf{y}=\mathbf{f}\left(x_1,x_2,...,x_n,B_1,B_2,...,B_k\right),\ i=1,2,...,m,\\ C_i<x_i<D_i,\ i=1,2,...,n.\end{cases} \quad (4)$$

The system of constraints (4) defines a certain curvilinear region in the space of primary factors $\mathbf{R}^n$ i.e. the region of product operability. Finding sets of the form (2) contained in it geometrically means inscribing n-dimensional parallelepipeds into this region. Such a problem has not a single solution, as there are countless such parallelepipeds that can be inscribed into the specified region. In this case, each of them can be completely defined by a point $\mathbf{x_0}=\left(x_{10},x_{20},...,x_{l0}\right)^T$, that is known to lie in this area and corresponds to one of the basic design variants, and a set of values of lower δ_i and upper Ω_i deviations of primary factors from their nominal values corresponding to the boundaries of the tolerance fields of primary factors, i.e. the selected technology:

$$x_{i0}-\delta_i\le x_{i0}\le x_{i0}+\Omega_i,\ i=1,2,...,n. \quad (5)$$

Not every solution (5) of the formulated problem can be practically realized due to various design, technological, economic or other considerations. The reason for this may be either high cost or lack of necessary equipment, components, materials, performers of appropriate qualification, peculiarities of the object design, etc. These restrictions can be analytically written down in the form of optimality criteria (target functions) of economic, production or other sense, expressed through deviations of primary factors from their nominal values:

$$F_i=F_i\left(\delta_1,\delta_2,...,\delta_n,\Omega_1,\Omega_2,...,\Omega_n\right),\ i=1,2,...,L,\ F\in\mathbf{R}^L,\ \delta,\Omega\in\mathbf{R}^n. \quad (6)$$

Obviously, of all the mentioned parallelepipeds, the most acceptable for practical implementation of the object are those in which criteria (6) or some of them will be optimized and the rest are attached to constraints (4).

Various criteria for optimizing the primary factor tolerances are possible. The main one is the minimizable cost function. Since the dependence of this function on the current values of the tolerances on each of the primary factors is unknown, one can try to replace it in some sense with equivalent criteria, for example, requiring the maximization of all or some of the tolerances. In this case, we will consider the tolerances on the values of the primary factors as partial optimality criteria:

$$F_i = -\delta_i \to \min,\ F_i = -\Omega_i \to \min,\ i = 1, 2, ..., n.$$

Thus, the problem of ensuring a given level of object quality is reduced to a multicriteria optimization problem with some constraints. It is required to determine such deviations δ_i и Ω_i from the nominal values of the primary factors at which the constraints (1) are fulfilled in the region of operability (5). The solution of the formulated problem is associated with some difficulties, the overcoming of which should become necessary solution points.

Ensuring stability of the mathematical model. The considered models are of practical importance only when the errors of the experimental input information cannot cause unacceptably large errors of the determined quantities, i.e., when the models are stable.

Let the system of equations of the type:

$$(\mathbf{A} + \Delta\mathbf{A})(\mathbf{x} + \Delta\mathbf{x}) = \mathbf{y} + \Delta\mathbf{y}, \tag{7}$$

where the elements of the matrix, the sought vector and the right part have some unknown absolute errors – $\mathbf{\Delta A}$, $\mathbf{\Delta x}$ and $\mathbf{\Delta y}$ respectively, depending on the accuracy of control and measuring equipment and other factors. In [3], the concept of model stability for all or a group of factors is defined, and an estimate of the relative error of the parameters identified using a linear model is derived:

$$\|\mathbf{\Delta x}\|/\|\mathbf{x}\| \le cond(\mathbf{A}) \cdot (\|\mathbf{\Delta y}\|/\|\mathbf{y}\|) + [cond(\mathbf{A})]^2 \cdot (\|\mathbf{\Delta A}\|/\|\mathbf{A}\|), \tag{8}$$

expressed in terms of the conditioning number $cond(\mathbf{A})$ of matrix $\mathbf{A}$ and errors of the measured characteristics and elements **A**. From (8) we can see that it is possible to regularize the model not only by influencing the operator **A**, which is not always possible in production conditions due to various reasons. One of the ways to narrow down the set of possible solutions to the correctness class is the method of multiple solution and finding the desired solution as the mathematical expectation of all obtained solutions. This method is a realization of the least squares method (LSM), and the obtained solutions are LSM estimates (LSE).

The described method of statistical solution is most effective in combination with methods of influence on the operator **A**, for example, the truncated estimation method. The method is based on involving the method of principal components as a linear filtering of least squares estimates. The essence of the filtering consists in such an action on ONC, which would significantly narrow the scattering ellipsoid of ONC by means of compression of the information contained in the scattering matrix due to “truncation” of the “tail” of the spectrum of the Fisher matrix.

Let instead of (7) we solve the equation:

$$\mathbf{Ax} = \mathbf{y} + \Delta\mathbf{y}, \tag{9}$$

where $\mathbf{y}$ is the true value; $\Delta\mathbf{y}$ is the vector of “noise” values, whose components are normally distributed $\Delta y_i \sim N(0, \sigma_i)$.

Then $\Delta\mathbf{y}$ is a multivariate normal variable with zero mean $\langle \Delta\mathbf{y} \rangle = 0$ and covariance matrix $\Sigma = \mathrm{cov}(\Delta\mathbf{y})$. Denoting the LSE as $\widehat{\mathbf{x}}$, we define its corresponding scattering matrix as $\mathbf{\Omega} = \left\langle (\mathbf{x} - \widehat{\mathbf{x}})^T (\mathbf{x} - \widehat{\mathbf{x}}) \right\rangle$. Given that $\mathbf{\Omega} = \mathbf{I}^{-1}$, where $\mathbf{I}$ I is the Fisher matrix, the spectral decomposition is:

$$\mathbf{I} = \mathbf{VDV}^T,\ \mathbf{D} = diag(\lambda_1, \lambda_2, ..., \lambda_n),\ \lambda_1 \geq \lambda_2 \geq ...\lambda_n > 0,$$

where $(\lambda_1, \lambda_2, ..., \lambda_n)$ are the eigenvalues of the Fisher matrix; $\mathbf{V}$ is an orthogonal matrix.

The columns of the matrix $\mathbf{V}$ – $(V_1, V_2, ..., V_n)$ define the directions of the principal axes of the ellipsoidal region of admissible estimates of the incorrectly posed problem (9). Let us decompose the LSE by the system of eigenvectors of the Fisher matrix as $\hat{\mathbf{x}} = \mathbf{V}\widehat{\mathbf{p}}$. Here $\widehat{p}_1, \widehat{p}_2, ..., \widehat{p}_n$ are the principal components of the LSE.

By cutting off the "tail" of the spectrum of the Fisher matrix, i.e., the smallest eigenvalues contributing the largest variance to the LSE, and choosing the required number of principal components ν after simple transformations we obtain a regularized solution in the form of a truncated estimator:

$$\mathbf{x}_{tr} = \mathbf{V}_{\nu}\mathbf{V}^{T}\hat{\mathbf{x}} . \quad (10)$$

Methods related to the optimization problem. A specific optimization method is selected for the problem to be solved from a rather large library of detailed optimization algorithms.

Statistical processing of empirical data. Due to many random situations during production and operation of products, as well as due to instability of properties of construction materials, their characteristics can be taken as random values. Then the realizations of these quantities can be used to obtain estimates of the true values, for example, by the method of confidence intervals, if the distribution laws are known [5, 6].

Analysis of results and conclusions

The proposed approach, formalizing in a general form the problem of optimal provision of requirements of specifications for output characteristics of a product or technological process, allows:

– establish the interrelation of the tasks of selecting the base variant of the object, determined by the nominal values of its primary factors, and assigning design and technological tolerances on them, based on the restrictions imposed in the TS on the output parameters of the object;

– to set and solve the problem of synthesizing design variants of products with optimal sensitivity to production and operational deviations of their primary factors, i.e. to link directly the choice of the basic variant of the object with the peculiarities of its practical realization;

– formalize a large number of important heterogeneous tasks of design, construction, production and testing regardless of the branch of engineering.

To verify the universality of the stated theory, the multiple inverse problem was set and solved for various branches of engineering: ensuring the strength and tightness of electronics elements [5]; reducing the vibration activity of gas turbine engines and turbopump units to a given level [6]; assigning reasonable tolerances for residual imbalances during the balancing and assembly of rotors; developing methods of balancing flexible rotors.

References

1. Gabor, T., Kiermeier, M., Sedlmeier, A., Kempter, B., Klein, C., Sauer, H., ... & Wieghardt, J. (2018). Adapting quality assurance to adaptive

systems: the scenario coevolution paradigm. In Leveraging Applications of Formal Methods, Verification and Validation. Distributed Systems: 8th International Symposium, ISoLA 2018, Limassol, Cyprus, November 5–9, 2018, Proceedings, Part III 8 (pp. 137–154). Springer International Publishing.

2. Kuwajima, H., Yasuoka, H., & Nakae, T. (2018). Open problems in engineering and quality assurance of safety critical machine learning systems. arXiv preprint arXiv:1812.03057.

3. Gallet, A., Rigby, S., Tallman, T. N., Kong, X., Hajirasouliha, I., Liew, A., & Smyl, D. (2021). Structural engineering from an inverse problems perspective Proc. R. Soc. A. 47820210526 20210526.

4. Goroshko, A. V., & Royzman, V. P. (2015). Statistical methods for providing the stability of the solutions of inverse problems and their application to decrease rotor vibroactivity. Journal of Machinery Manufacture and Reliability, 44, 232–238.

5. Goroshko, A.V., Royzman, V.P., Bubulis, A., Juzėnas, K. Methods for testing and optimizing composite ceramics-compound joints by solving inverse problems of mechanics // Journal of Vibroengineering. Vol. 16. Issue 5, 2014, P. 2178–2187.

6. Goroshko, A. V., Royzman, V. P. Barmina, O. V. Study of the stability and accuracy of the eccentricity identification algorithm in fast-revolving rotor balancing problems. Journal of Machinery Manufacture and Reliability 45 (2016): 227–232.

KINEMATIC ANALYSIS OF DWELL LINKAGE MECHANISMS USING SOLIDWORKS MOTION

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The problem of designing of mechanisms that can provide a prescribed dwell of the output link, during the continuous rotation of the input link, is an important practical task, since these mechanisms are widely used in the design of various modern machines. For this purpose, different types of mechanisms can be used, in particular cam mechanisms, but, as it is known [1–7], in many cases linkage mechanisms have a number of essential advantages due to the absence of higher kinematic pairs and the presence of geometric closure of the links. Thus, they are practically more reliable and durable, and these mechanisms are able to provide higher operating speeds of machines, which is especially important for automatic machines.