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## **ANALYSIS OF UNBALANCED INDUCTION MOTOR ROTOR FORCES IN BEARINGS WITH CLEARANCES**

**Abstract.** The paper analyzes the influence of unbalanced rotor forces of induction motor on the operation of bearings with clearances. A mathematical model considering mechanical unbalance and static magnetic unbalance is developed in the paper. The results of modeling showed that the joint action of these factors leads to unstable operation mode of bearings and accelerated wear. The unbalance factor depends on the static eccentricity angle and the instantaneous value of the trunnion rotation angle. The obtained results can be used to optimize the design of induction motors and to develop methods of diagnostics of their condition.

**Keywords:** Induction Motor, Eccentricity of rotor mass, bearing fault, Unbalanced Magnetic Pull

### **Introduction**

The problem of vibration reliability of electric motor bearings continues to be topical. Various methods are proposed for its solution: active and passive autobalancing devices [1], electromagnetic suspensions [2], rational design of motors [3], modeling and vibration analysis of bearing units [4].

For induction motors (IM), bearings mounted in special risers are used as rotor supports. The risers are bolted to the lower half of the end shield. For AMs, bearing failures account for about 40% of the total number of induction machine failures [5, 6]. Bearing failure is mainly caused by their increased wear. The main factors of wear are mechanical imbalance of the rotor due to the eccentricity of the rotor mass and unbalanced magnetic pull (UMP) due to magnetic eccentricity. The main causes of rotor eccentricity in IMs are bearing damage, misalignment of motor and load, loosening of motor mountings and other factors.

The action of rotor unbalance and magnetic unbalanced forces is manifested by taking into account the bearing clearances, which are summarized by elastic deformations and passport clearance. During the operation of the motor, an additional increase in static magnetic eccentricity is possible due to the increase in bearing clearance caused by dynamic load [7]. The purpose of this work is to analyze the joint influence of unbalanced forces of mechanical and electromagnetic nature on the operating mode of rotor bearings.

### Methodology

Radial bearings are characterized by the amount of clearance between journal and shell or between inner and outer rings. The amount of clearance affects the movement of the journal and the behavior of the unbalanced rotor.

In general, three systems of forces act on the trunnions of an unbalanced rotor.

1. Static forces that are constant in magnitude and direction. In general, static forces can be defined as

$$\vec{Q} = \vec{G} + \vec{F}, \quad (1)$$

where  $\vec{G}$  is the rotor weight;  $\vec{F}$  is the radial component of forces from belt, chain, gear wheel, etc.

2. Dynamic unbalanced forces that are constant in magnitude but variable in direction. These are forces caused by mass unbalance of the rotor:

$$F_e = me\omega^2, \quad (2)$$

where  $e$  is the eccentricity of the rotor mass  $m$ ,  
 $\omega$  is the rotor angular speed.

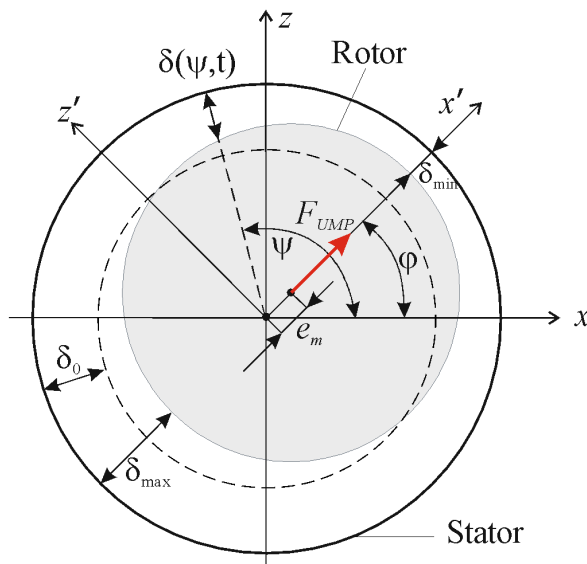
3. Dynamic forces variable in magnitude and direction, such as unbalanced magnetic pull forces  $\vec{F}_{UMP}$  caused by magnetic eccentricity.

The level of unbalance of the rotor in the plane of each bearing can be estimated by the unbalance coefficient (unbalance factor), which is equal to the ratio of the dynamic forces to the static forces:

$$K_{12} = \frac{|F_e + F_{UMP}|}{Q} \quad (3)$$

At  $K < 1$  the mode of one-sided bearing wear is observed; at  $K = 1$  the most dangerous mode of periodic impacts of the rotor on the bearing surface is observed; at  $K > 1$  the mode of one-sided wear of trunnions is observed, at which the rotor rolls the bearing surface.

To estimate the force  $F_{UMP}$ , caused by UMP we use the theoretical expressions proposed in [6, 8] (Fig. 1).



**Fig. 1.** Diagram of magnetic eccentricity of the rotor

The value of the air gap of an eccentric rotor for an arbitrary eccentricity at an arbitrary time is approximately equal:

$$\delta(\psi, t) \approx \delta_0 - e_m \cos(\psi - \phi) \quad (4)$$

where  $\delta_0$  is the average size of the air gap when centering the rotor,  $\psi$  is the stator angle,  $e_m$  is the value of magnetic eccentricity,  $\phi$  is the eccentricity position angle (rotation of the rotor center).

With a dynamic eccentricity and a constant rotational speed  $\omega = \text{const}$  the eccentricity angle is a function of time  $\phi = \phi_0 + \omega t$ , so

$$\delta(\psi, t) \approx \delta_0 - e_m \cos(\psi - \phi_0 - \omega t), \quad (5)$$

where  $\phi_0$  is the initial angle of the eccentricity position,

$\omega$  is the angular speed of rotor rotation.

The main magnetomotive force of rotor excitation (MMF) is defined as:

$$F(\psi, t) = F_j \cos(\omega_e t - p\psi) \quad (6)$$

where  $\omega_e$  is the angular frequency of the power supply to the motor stator windings,

$$\omega_e = \frac{\omega p}{1-s};$$

$p$  is the number of pole pairs of stator;

$s$  is the motor slip.

The magnetic permeability of the air gap  $\Lambda(\psi, t)$  is expressed using the Fourier series

$$\Lambda = \frac{\mu_0}{\delta(\psi, t)} = \sum_{i=0}^{\infty} \Lambda_i \cos i(\psi - \varphi),$$

where 
$$\Lambda_i = \frac{\mu_0}{\delta_0} \frac{1 + (1 - \delta_{i0})}{\sqrt{1 - \varepsilon^2}} \left( \frac{1}{1 + \sqrt{1 - \varepsilon^2}} \right)^i, \quad i \geq 0.$$

(7)

where  $\varepsilon = \frac{e_m}{\delta_0}$  indicates relative eccentricity;

$\mu_0$  is the absolute magnetic permeability.

$\delta_{i0}$  is Kronecker symbol.

UMP is highly nonlinear. Non-linear methods for calculating UMP use the energy method or the integration of the Maxwell stress tensor in the air gap between the stator and rotor. The expression for the complex force of unbalanced magnetic attraction on the rotor surface is defined as

$$\underline{F}_{UMP} = f_1 e^{i\varphi} + f_2 e^{i(2\omega_e t - \varphi)} + f_3 e^{i(2\omega_e t - 3\varphi)}, \quad (8)$$

where  $f_1, f_2, f_3$  is the the amplitudes of the UMP components, which are found by the formulas:

$$\begin{aligned}
f_1 &= \frac{Rl\pi}{4\mu_0} F_j^2 (2\Lambda_0\Lambda_1 + \Lambda_1\Lambda_2 + \Lambda_2\Lambda_3), \\
f_2 &= \frac{Rl\pi}{4\mu_0} F_j^2 \left( \Lambda_0\Lambda_1 + \frac{1}{2}\Lambda_1\Lambda_2 + \frac{1}{2}\Lambda_2\Lambda_3 \right), \\
f_3 &= \frac{Rl\pi}{4\mu_0} F_j^2 \left( \Lambda_0\Lambda_3 + \frac{1}{2}\Lambda_1\Lambda_2 \right),
\end{aligned} \tag{9}$$

where  $R$  is the rotor radius,

$l$  is the rotor length,

$F_j$  is the MMF amplitude (6).

It is convenient to represent the force  $\overset{\omega}{F}_{UMP}$  in complex form as the sum of the static magnetic component  $\overset{s}{F}_{UMP} = f_1 e^{i\varphi}$  and the dynamic component  $\overset{d}{F}_{UMP} = (f_2 e^{-i\varphi} + f_3 e^{-i3\varphi}) e^{i(2\omega_e t)}$ . Then expression (3) can be represented as

$$K = \left| \frac{\overset{\omega}{F}_{\overset{s}{M}} + \overset{\omega}{F}_{\overset{d}{M}UMP}}{\overset{\omega}{Q} + \overset{s}{F}_{UMP}} \right| \tag{10}$$

### Numerical example

To simplify the analysis, let us concretize the problem and consider one of the trunnions of a squirrel cage induction motor with a power of 11 kW with a squirrel-cage rotor with a permissible level of vibration displacement of 70 microns. The main characteristics of the motor are shown in Table 1. The rotor bearings are 6208 ZZ-C3 ball bearings with a clearance of 25 microns, mounted in cast iron bearing plates.

Let's consider further the regime of one-sided bearing wear, which is observed at the value of  $K < 1$ , taking into account the rotor mass eccentricity and magnetic eccentricity.

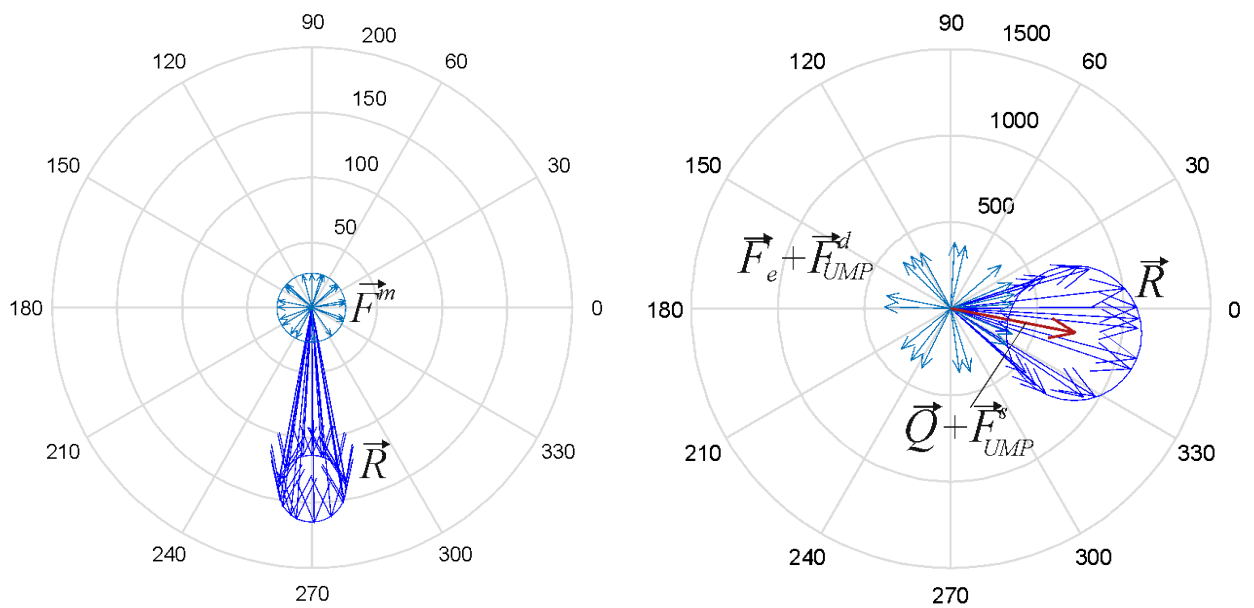
Table 1. Parameters of the three-phase motor

Notatio	Description	Value
$n$		
$n$	Rated speed (rpm)	3000
$s$	Rated slip	0.033
$R$	Radius of the rotor (mm)	63.5
$m_r$	Mass of the rotor (kg)	14.22
$\delta_0$	Mean air-gap length (mm)	0.45

$\mu_0$	Air permeance (H/m)	$4\pi \cdot 10^{-7}$
$F_j$	Fundamental MMF amplitude of the rotor excitation current (A)	945
$p$	Number of pole pairs	1
$e$	Permissible eccentricity ( $\mu\text{m}$ )	20
$d$	Bearing inner diameter (mm)	40

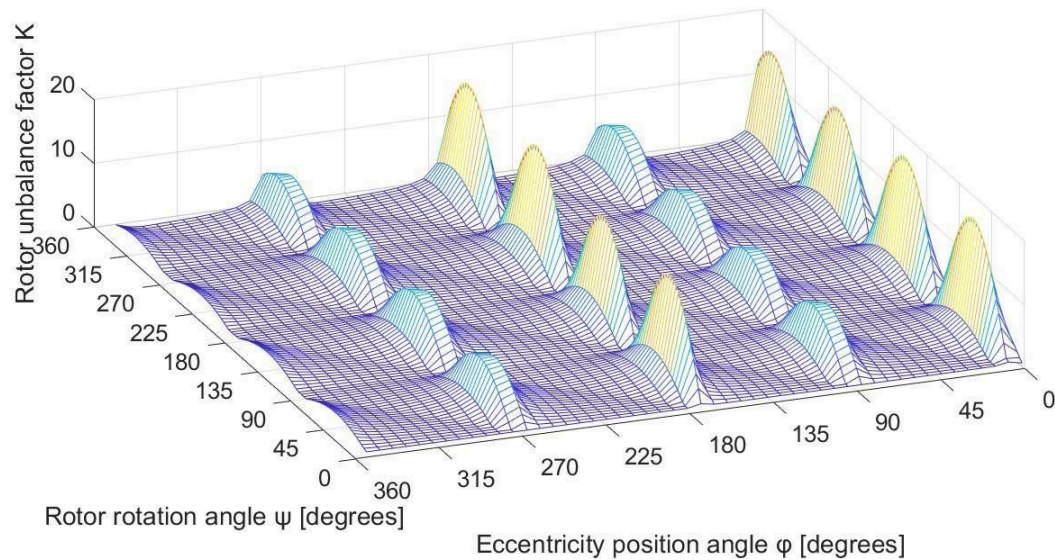
Consider the effect of mass eccentricity without taking into account the magnetic eccentricity. In this case, for an allowable eccentricity of  $e=20 \mu\text{m}$ , at the rotor operating speed  $\omega=303,7 \text{ c}^{-1}$  we have an unbalance coefficient  $K=0,19$ . Then the hodograph of the equivalent force  $\vec{R}$  will be a circle of radius  $R$  ( $Q - F_e \leq R \leq Q + F_e$ ), as shown in Fig. 2. The center of the journal oscillates pendulum-like with an angular amplitude approximately equal to  $\alpha=11^\circ$ . The journal is worn around the entire circumference and the bearing within an arc of length  $L = d\alpha = 7.7 \text{ mm}$ .

Consider the action of magnetic static eccentricity  $\varepsilon=0.01$ , located at an angle  $\phi=0^\circ$ . Godographs of dynamic and resultant forces acting on the bearing are shown in Fig. 3. In this case, the unbalance factor  $K$  depends on the static eccentricity angle  $\phi$  and the instantaneous value of the trunnion angle  $\psi$ , as shown in Fig. 4. Analysis of the dependence shows that the unbalance coefficient  $K$  at certain eccentricity positions can take values  $K<1$  and  $K>1$ . Such an unstable operating mode of bearings negatively affects their operational properties and can significantly reduce their service life.



**Fig. 2.** Godographs of dynamic force  $\overset{\omega}{F}_e$  and the resultant force  $\overset{\omega}{R}$  when the rotor mass imbalance is taken into account

**Fig. 3.** Godographs of dynamic and resultant forces when mass and UMP imbalances are taken into account



**Fig. 4.** Dependence of the unbalance coefficient  $K$  on position angles of eccentricity and rotor rotation

## Conclusions

The first bearing operation mode, when the rotor axis moves along the arc of a circle with radius equal to half of the diametral clearance under the action of mass unbalance on the rotor, can be achieved at the unbalance coefficient  $K < 1$ .

At action of static magnetic eccentricity of the rotor the account of dynamic magnetic forces essentially complicates a problem of definition of a mode of operation of the bearing. In this case, the unbalance coefficient  $K$  depends on the static eccentricity angle  $\phi$  and the instantaneous value of the trunnion rotation angle  $\psi$ .

The obtained results can be used to optimize the design of induction motors and to develop methods of diagnostics of their condition.

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