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Metaphysical Modeling of Rotor dynamics Motor Spindle with magnetic Eccentricity

Abstract. The paper presents a metaphysical model of the rotor dynamics of an asynchronous motor spindle that accounts for the effect of magnetic eccentricity. The study proposes a new approach to modeling unbalanced magnetic pull (UMP) based on a variable mutual inductance of the magnetizing circuit instead of the classical Maxwell stress tensor method. Analytical dependences between the air-gap eccentricity, magnetizing inductance, and UMP components are derived, and a modified Simscape/Simulink model of the asynchronous machine is implemented to simulate these relationships. The simulation results demonstrate the influence of magnetic eccentricity on the electromagnetic torque, stator current spectrum, and rotor speed. The proposed model provides a practical tool for diagnosing eccentricity-related faults and predicting vibration behavior in motor spindles.
Keywords: motor spindle, vibrations, UMP, eccentricity, Seascape.

1.Introduction

The failure of the bearings of asynchronous machines is mainly caused by their moving wear. The main factors in the wear of such bearings are their imbalance, eccentricity of the rotor mass (mechanical imbalance) and magnetic eccentricity (UMP). Fragments of asynchronous motors leave a relatively small impact on the gaps, but the smell is more similar to UMP. Today, the power supply dynamics of asynchronous motors are actively monitored [1-3]. Particularly, it is possible to place a low level of vibration on the motor spindles; the excess of the permissible vibrations does not negatively affect the service life of the motor, and reduces the accuracy of machining on the bench. Therefore, to combat vibrations, it is important to have accurate and efficient models of the motor spindle dynamics.

1. Related works

Due to high to low vibration levels of motor spindles, the problem of vibration reduction and control remains relevant. The robot [4] examined the numerical distribution of the alignment of the rotor system with unbalance and defects in the connection. In work [5], the dynamics of an eccentric motor-spindle was studied, for which a model was created that combines magnetic eccentricity and mass eccentricity. The authors of [6] observed connections between thermal expansion and the magnitude of the wind gap, which triggers the appearance of UMP and frequency-dependent nonlinear changes in hardness. Bearing faults in the mechanical system run by an induction motor causes change in its stator current spectrum. The faults in the bearings cause variations of load irregularities in the magnetic field which in turn change the mutual and self-inductance causing side bands across the line frequency [7]. Article [8] compares the bearing fault detection capabilities obtained using vibration and current signals. The article utilizes a simple and effective method for processing current and vibration signals, as well as a theoretical analysis of the physical relationship between faults modeled by torque disturbances and current components. The article focuses on the theoretical justification for the correlation between torque disturbances and the amplitude of current components.

In most models, the UMP expansion is based on the energy method or integration of the Maxwell stress tensor at the wind gap between the stator and rotor, the magnetic penetration of the gap is distributed in the Fourie series. The purpose of this article is to create a mathematical model for the rotation of the eccentric rotor of the asynchronous spindle motor, so as to determine the values of the variable mutual inductance of the motor. The very change in inductance due to an uneven gap is the reason for the appearance of UMP. In addition, changing the inductance affects the electrical and mechanical characteristics of the motor spindle.

3, Mathematical modeling of UMP

In the general case, a rotor with mass m and mass eccentricity e and magnetic eccentricity e_m, rotating at speed ω, is subjected to radial unbalance forces $e\omega^2 m$ and F_{UMP} (Fig. 1). The uneven air gap caused by magnetic eccentricity results in an unbalanced magnetic tension (Fig. 2).

With static magnetic eccentricity, the air gap is a function of the rotor rotation angle ψ

$$\delta(\psi,t)=\delta_0-e_m\cos(\psi-\varphi)=\delta_0\left(1-\varepsilon\cos(\psi-\varphi)\right), \quad (1)$$

where δ_0 – average air gap when centering the rotor;

ψ - angular coordinate on the rotor circle;

e_m – magnetic eccentricity value;

$\varepsilon = \dfrac{e_m}{\delta_0}$ – value of relative eccentricity;

φ - eccentricity position angle (static rotor eccentric angle).

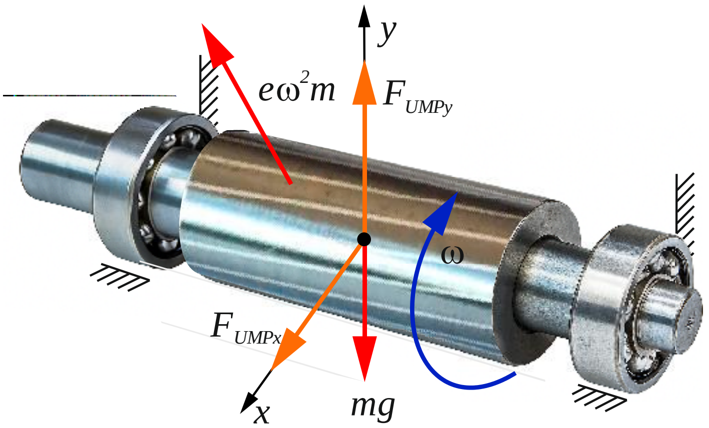

Fig. 1. Unbalanced forces acting on an eccentric rotor

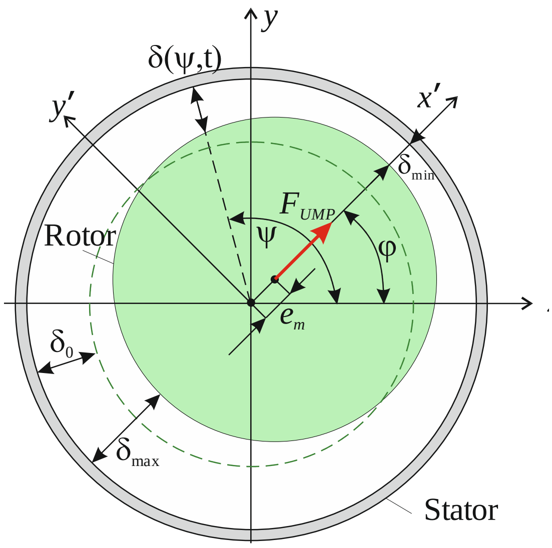

Fig. 2. Scheme of formation of magnetic eccentricity of the rotor and F_{UMP}

The magnetic permeability of the gap is a function of the rotation angle and the relative eccentricity:

$$\Lambda(\psi,\varepsilon) = \frac{\mu_0}{\delta(\psi,t)} = \frac{\mu_0}{\delta_0\left(1-\varepsilon\cos(\psi-\varphi)\right)}. \tag{2}$$

The inductance of the magnetizing $L_m(\psi,\varepsilon) \sim \Lambda(\psi,\varepsilon)$ circuit depends on the number of turns and the cross-sectional area of the magnetic core. Then, taking into account (2), we have

$$L_m(\psi,\varepsilon) = \frac{L_{m0}}{1-\varepsilon\cos(\psi-\varphi)} \tag{3}$$

where L_{m0} - gap inductance in the absence of eccentricity.

Dependence (3) is approximate, since it does not take into account: the phenomenon of saturation of the magnetic core, the influence of teeth and grooves, considering the stator and rotor as absolutely smooth cylinders. The scattering of the magnetic field outside the magnetic core is not taken into account. The magnetic permeability is considered infinitely large and all the magnetic resistance is concentrated in the air gap.

With dynamic eccentricity and constant rotor rotation speed ω=const, initial angle of position of dynamic eccentricity φ_0, the angle of position of eccentricity is a function of time defined as $\varphi(t)=\omega t+\varphi_0$. In this case, mutual inductance $L_m(\varepsilon,t)$ for a given eccentricity ε is a function of time and the angle of rotation of the rotor (4). The instantaneous value of eccentricity projections is defined as $e_x=e_m\cos(\omega t+\varphi_0)$, $e_y=e_m\sin(\omega t+\varphi_0)$.

The UMP force is the derivative of the accumulated magnetic energy along the displacement coordinate e_m. To account for the effect of all the energy at each point in the gap, we apply integration over the entire circle:

$$F_{UMP}=-\frac{\partial W}{\partial e_m}=-\frac{1}{2}i_m^2(t)\int_0^{2\pi}\frac{\partial L(\psi,t)}{\partial e}d\psi\ , \tag{4}$$

where $i_m(t)$ - instantaneous value of magnetizing current.

The projections of the force UMP on the coordinate axis, along which the displacement of the dynamic eccentricity e_m occurs during rotor rotation, is calculated a

$$\begin{aligned} F_{UMPx}&=F_{UMP}\cos\varphi(t)=-\frac{\pi L_{m0}i_m^2(t)\varepsilon}{\delta_0\left(1-(\varepsilon)^2\right)^{\frac{3}{2}}}\cos(\omega t+\varphi_0)\\ F_{UMPy}&=F_{UMP}\sin\varphi(t)=-\frac{\pi L_{m0}i_m^2(t)\varepsilon}{\delta_0\left(1-(\varepsilon)^2\right)^{\frac{3}{2}}}\sin(\omega t+\varphi_0) \end{aligned} \tag{5}$$

Taking the instantaneous value of the magnetizing current with a cyclic frequency ω_e,

$$i_m=I_m\cos\omega_e t\ ,$$

we have

$$\begin{aligned} F_{UMPx}&=-\frac{\pi\varepsilon L_{m0}I_m{}^2}{\delta_0\left(1-\varepsilon^2\right)^{\frac{3}{2}}}\Big(2\cos(\omega t+\varphi_0)+\cos\big((2\omega_e-\omega)t-\varphi_0\big)+\cos\big((2\omega_e+\omega)t+\varphi_0\big)\Big),\\ F_{UMPy}&=-\frac{\pi\varepsilon L_{m0}I_m{}^2}{\delta_0\left(1-\varepsilon^2\right)^{\frac{3}{2}}}\Big(2\sin(\omega t+\varphi_0)+\sin\big((2\omega_e+\omega)t+\varphi_0\big)-\sin\big((2\omega_e-\omega)t-\varphi_0\big)\Big). \end{aligned} \tag{6}$$

Considering the special case of only static eccentricity at φ=const, we have:

$$F_{UMPx} = -\frac{\pi\varepsilon L_{m0} I_m{}^2}{\delta_0 \left(1-\varepsilon^2\right)^{\frac{3}{2}}} \left(2\cos\varphi + \cos\left(2\omega_e t + \varphi\right) + \cos\left(2\omega_e t - \varphi\right)\right),$$
$$F_{UMPy} = -\frac{\pi\varepsilon L_{m0} I_m{}^2}{\delta_0 \left(1-\varepsilon^2\right)^{\frac{3}{2}}} \left(2\sin\varphi + \sin\left(2\omega_e t + \varphi\right) - \sin\left(2\omega_e t - \varphi\right)\right). \quad (7)$$

Therefore, due to static magnetic eccentricity, the UMP force has a constant component directed towards the smallest gap size $\delta_{\min}$ and a component pulsating at double the electrical frequency. With dynamic eccentricity, the constant component is converted into a harmonic at the rotor speed.

The developed model was implemented on the basis of the standard Asynchronous Machine block from the Simulink/Simscape library. To describe the dynamic processes in an asynchronous machine, a flat orthogonal coordinate system *dq*, is used here, which rotates at a speed of ω. The electrical part of the engine is described by a fourth-order system, and the mechanical part by a second-order system. All electrical variables and parameters refer to the stator. The system of differential equations describing the dynamics of an asynchronous machine when changing voltage, currents, torques and speeds has the form [9]:

$$\begin{aligned} U_{qs} &= R_s i_{qs} + \frac{d\psi_{qs}}{dt} + \omega_e \psi_{ds}, \\ U_{ds} &= R_s i_{ds} + \frac{d\psi_{ds}}{dt} - \omega_e \psi_{qs}, \\ U'_{qr} &= R'_r i'_{qr} + \frac{d\psi'_{qr}}{dt} + \left(\omega_e - \omega_r\right)\psi'_{dr}, \\ U'_{dr} &= R'_r i'_{dr} + \frac{d\psi'_{dr}}{dt} - \left(\omega_e - \omega_r\right)\psi'_{qr}, \end{aligned} \quad , \quad \begin{aligned} M_e &= 1.5p\left(\psi_{ds} i_{qs} - \psi_{qs} i_{ds}\right), \\ \psi_{qs} &= L_s i_{qs} + L_m i'_{qr}, \\ \psi_{ds} &= L_s i_{ds} + L_m i'_{dr}, \\ \psi'_{qr} &= L'_r i'_{qr} + L_m i_{qs}, \\ \psi'_{dr} &= L'_r i'_{dr} + L_m i_{ds}, \\ L_s &= L_{ls} + L_m, L'_r = L'_{lr} + L_m. \end{aligned} \quad , \quad (8)$$

where $U_{qs} = U_m \cos\omega t$, $U_{qs} = U_m \sin\omega t$ - voltages on the stator windings along the *q* and *d* axes, respectively;

U'_{qr}, – rotor voltage;

i_{qs}, i_{ds} – stator currents;

i'_{qr}, i'_{dr} – rotor currents;

ψ_{qs}, ψ_{ds} – stator flux linkage projections;

$d\psi'_{qr}, d\psi'_{dr}$ – rotor flux coupling projections;

L_s – total stator inductance;

L_{ls} – stator leakage inductance;

L'_r – total rotor inductance;

L_m – inductance of the magnetizing circuit;

ω_r – electrical angular velocity of the rotor;

M_e – electromagnetic torque of the motor.

In the basic model, the Mutual inductance L_m=const parameter is entered once at the beginning of the simulation and is not expected to change during the simulation. In order to implement a variable magnetization inductance in the model, the basic Asynchronous Machine block was modified. For this purpose, a mask adjustment was performed, which made it possible to replace the fixed parameter L_m with a variable one $L_m(\psi, t)$, which is supplied from the outside. This provided dynamic control of the parameter during the simulation and calculation of all electrical and mechanical parameters when the inductance of the magnetization circuit changes and more accurate calculations of the electrical and mechanical parameters of the motor.

4.Simulation Results and Discussion

The dynamic model of the rotor of a motor-spindle with an eccentric mass and unbalanced magnetic tension, described by equations (6) and (8), was implemented as a simulation Simscape model and solved numerically based on MATLAB. The object of the study was the spindle STF80-2.2-ER20 with an average air gap when the rotor is centered at 0.2 mm. The time dependences of the magnetizing inductance for different eccentricities are presented in Fig. 3.

The presence of magnetic eccentricity causes the appearance of harmonics with double frequency in the stator supply current spectrum (Fig. 5). The amplitude spectrum of the stator current for different eccentricities around the frequency $2\omega_e$ is presented in Fig. 6.

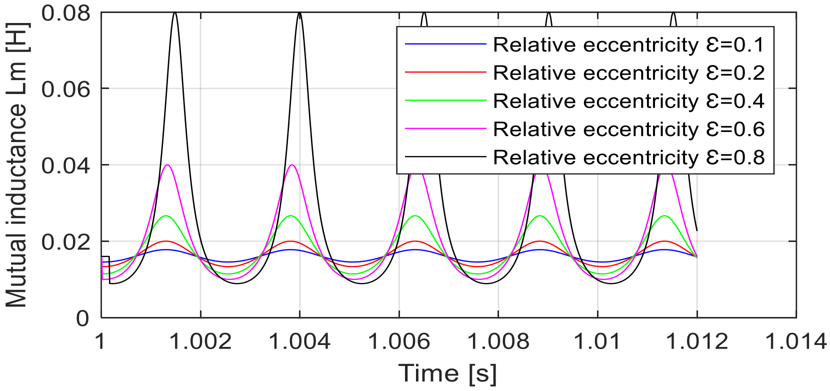

Fig. 3. Time dependences of magnetization inductance for different eccentricities

Analysis of the obtained results showed that at a nominal rotation speed of 24,000 rpm and a corresponding electric current frequency of 400 Hz, dynamic magnetic eccentricity causes minor fluctuations in speed, electromagnetic torque (Fig. 4), and rotor power with the rotor rotation frequency.

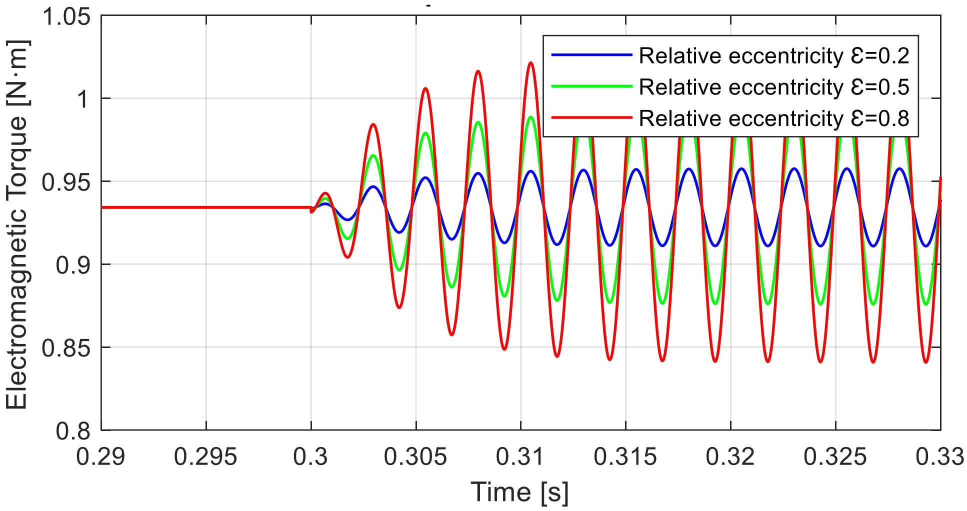

Fig. 4. Time dependences of the electromagnetic moment for different values of eccentricities

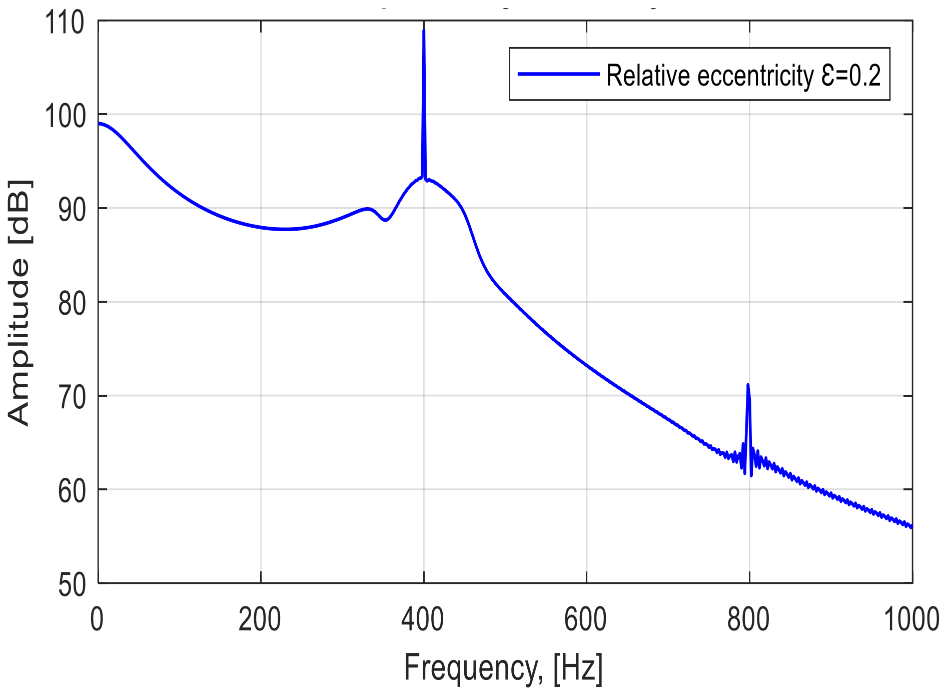

Fig. 5. Amplitude spectrum of stator current for eccentricity ε=0.2

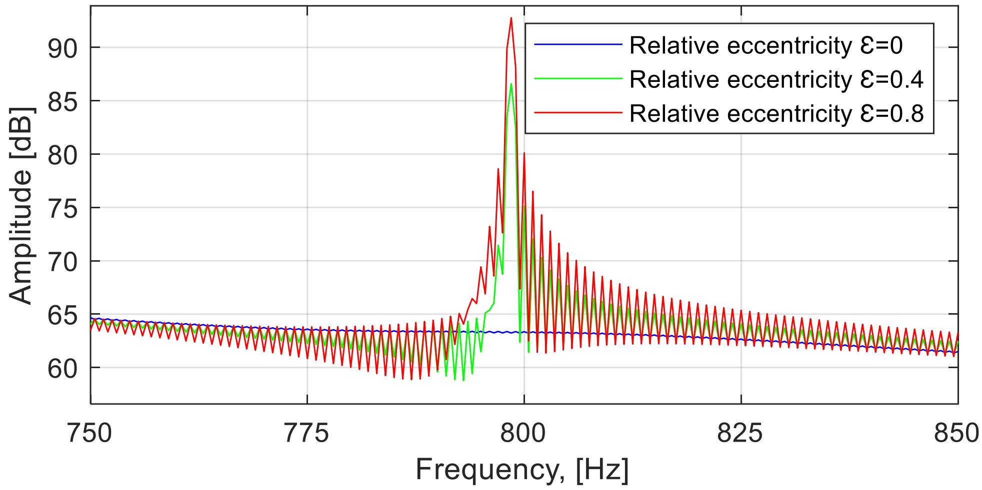

Fig. 6. Amplitude spectrum of stator current for different eccentricities

5. Conclusion

In contrast to the generally accepted approach to modeling UMP using Maxwell's equations and Fourier series expansion of the magnetic permeability of the gap, this work proposes to model UMP using a variable mutual inductance of the motor. For this purpose, in the case of dynamic magnetic eccentricity, analytical dependences of the magnetizing inductance, the dependence of UMP on the magnetizing inductance, and the standard Asynchronous block in Simulink/Simscape were obtained. This approach makes it possible to build Multiphysics models of arbitrary complexity based on the standard Simscape library. The proposed approach to modeling vibrations takes into account not only the influence of UMP, but also the influence of variable magnetizing inductance on the electrical and mechanical parameters of the motor.

The simulation results confirm the insignificant influence of the permissible eccentricity on the stator current, shaft torque and rotor speed. The obtained dependences for describing the UMP forces on the position of the rotor eccentricity and magnetizing inductance are consistent with the results obtained by other authors using the energy method and the method of integrating Maxwell's equations. The results obtained in this article are useful for diagnosing faults and predicting the dynamic eccentricity and unbalance of the motor-spindle.

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