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Movement of Working Fluid in the Field of Centrifugal Forces and Forces of Weight

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1. Introduction

For machines with variable imbalance of rotor and during the rotor balancing machines without stopping in operating conditions, the traditional methods of reducing vibration are ineffective. These machines are separators and centrifuges used in various sectors of the economy (food, chemical, sugar, mining, etc.), medicine and in everyday life. The most reliable, promising and often only one possible method of reducing vibration of these machines is the automatic balancing by using units with free movement of corrective masses.

This article analyses the work of automatic balancing units (self-balancing) having the form of a hollow chamber partially filled with a working fluid and being passive regulators of direct action that does not require power inlet and control system for corrective movement of the masses.

The aim of this work is to analyze the movement of fluid in a cylindrical chamber of ABU for machine rotors with a horizontal axis of rotation.

2. Fluid motion without the damping

In many cases, liquid auto -balancing units of direct action operate on rotors with a horizontal axis of rotation. This liquid is subjected to simultaneous action of centrifugal force and gravity fields. To determine the relative nature of the fluid in these conditions, consider a cylindrical chamber rotating with angular velocity ω = const, partially filled with fluid (fluid imbalance less than the rotor imbalance). The distance of O from the axis of rotation O_1 (deflection) is f (Fig. 1).

Suppose that liquid deflected from the equilibrium position (straight OA) at an angle α and is by itself. Construct a differential equation α changes over time. We apply the Angular Momentum to the fluid, taking momentum axis of rotation as the axis of fluid passing through the point O. Liquid is involved in two movements - with a portable camera and a relative - at the center of rotation O. The angle between the direction O_1C and equilibrium is θ. To the center of mass C of the liquid applied five forces: gravity $\overline{G} = m\overline{g}$ (m – of liquid), directed vertically downward along the line OC normal component of inertial forces $\overline{N}$ and Coriolis force of inertia $\overline{K}$, the tangential component of the force of inertia $\overline{T}$, aims perpendicular to the line OC, and directed along the instantaneous radius r, the centrifugal force of inertia relative motion $\overline{F}$. Moments Coriolis force $\overline{K}$ and $\overline{N}$ component relatively to point O is zero.

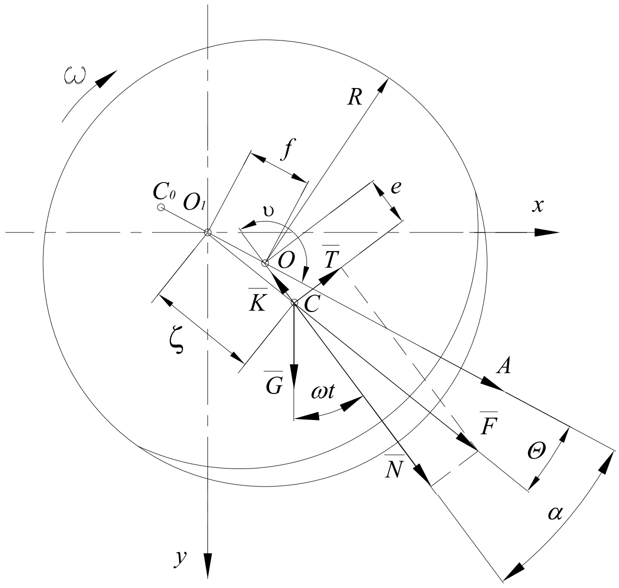

Fig. 1 Fluid in the field of centrifugal forces and gravity

The differential equation of the relative motion of the liquid is

$$me^2\ddot{\alpha} + \left(mre\omega^2\cos\theta - mge\cos\omega t\right)\sin\alpha = 0 \qquad (1)$$

where: $r^2 = f^2 + e^2 + 2fe\cos\alpha$, $\theta = \arcsin(\frac{e}{r}\cos\alpha)$; $e = R_b f/(R^2\text{-}f^2)$; $m \approx 2Rl\rho_l$; R, R_b - radii camera and free liquid surface at $f = 0$; ρ_l - liquid density.

Equation (1) essentially nonlinear, so consider the case of small oscillations of the liquid when $\sin\alpha \approx \alpha$; $\cos\alpha \approx 1$; $\cos\theta \approx 1$. Then from (1) we obtain an approximate equation in the form

$$me^2\ddot{\alpha} + me\left(r\omega^2 - g\cos\omega t\right)\alpha = 0 . \qquad (2)$$

After substituting in equation (2) $r = f + e$, we obtain.

$$\ddot{\alpha} + \omega^2\left[\frac{(f+e)}{e} - \frac{g\cos\omega t}{e\omega^2}\right]\alpha = 0 \qquad (3)$$

or

$$\ddot{\alpha} + \omega^2 \frac{f+e}{e}\left[1 - \frac{g}{(f+e)\omega^2}\cos\omega t\right]\alpha = 0 .$$

Turning to the dimensionless time $2\tau=\omega t$, have,

$$\frac{d^2\alpha}{dt^2} = \frac{\omega^2}{4}\cdot\frac{d^2\alpha}{d\tau^2},$$

equation (3) becomes the standard form of Mathieu equation [1]

$$\frac{d^2\alpha}{d\tau^2} + (b - 2\varepsilon\cos 2\tau)\alpha = 0 , \tag{4}$$

where:

$$b = 4\frac{f+e}{e},\ \varepsilon = 2\frac{g}{e\omega^2}. \tag{5}$$

Solutions of Mathieu equation are oscillatory in nature and depend on the values of b and ε. In some cases, this combination of b and ε correspond with limited fluctuations, while others - with increasing amplitudes. The boundaries between areas of stable and unstable oscillations determined from Eins Strett diagram [1]. The system is stable if the point due coordinates b and ε, is located inside the shaded area of the diagram. All other points correspond to unstable systems.

From (5) is shown that the parameter b depends on the deflection of rotor f in the balancing can range from $f_{\max}$ to $f_{\min}$, where $f_{\max}$ corresponds to the limit value of the rotor imbalance $D_{\max}$ and $f_{\min}$ - valid imbalance. The parameter ε is inversely proportional to the angular velocity ω and with its growth it decreases.

To stabilize the liquid inequalities should be performed

$$\begin{cases} b < 4 - \dfrac{\varepsilon^2}{12}; \\ b > 4 + \dfrac{5\varepsilon^2}{12}. \end{cases}$$

Or after substituting values b, ε and transformation

$$\begin{cases} |-f| > \dfrac{g^2}{12e\omega^4}; \\ f > \dfrac{5g^2}{12e\omega^4}. \end{cases}$$

The last system gives values for the remaining rotor axis deviation f_{res} created by residual imbalance of $D_{\text{res}} \le D_{\text{accept}}$. The first condition corresponds to an equilibrium position in subcritical fluid rotation mode, and the second condition - supercritical rotation mode.

For the full balance of the rotor (f=0) parameter b=4 and the successive states of the system in the diagram coincide with the vertical line passing through the point with coordinate b=4 and correspond to unstable solution of equation (4).

3. Fluid motion with regard to damping

If the system is entered the damping, the equation (4) takes the form

$$\frac{d^2\alpha}{d\tau^2} + 2h\frac{d\alpha}{d\tau} + (b - 2\varepsilon\cos 2\tau)\alpha = 0 \ (h>0), \tag{6}$$

where: $h = \dfrac{n}{2a}$, n – coefficient of viscosity, a – inertia factor.

Substituting in equation (6) $\alpha = q\cdot e^{-h\tau}$ gives

$$\frac{d^2q}{d\tau^2} + (b - h^2 - 2\varepsilon\cos 2\tau)q = 0 . \tag{7}$$

If $b_1=b-h^2$ (7) coincides with Mathieu equation (4).

In general, friction has stabilizing effect and leads to some narrow instability. However, as it is well known [2], the damping is unable to limit the increase in amplitude of these areas.

If the values are b_1, h, ε equation (7) defines a relatively stable movement q, that according to (6) motion asymptotically stable relative to α. Then the stability condition of the liquid for selected parameters b, h, ε, will be written in the form

$$\begin{cases} b - h^2 < 4 - \dfrac{\varepsilon^2}{12}; \\ b - h^2 > 4 + \dfrac{5\varepsilon^2}{12}; \end{cases} \text{ or } \begin{cases} b < 4 - \dfrac{\varepsilon^2}{12} + h^2; \\ b > 4 + \dfrac{5\varepsilon^2}{12} + h^2. \end{cases}$$

Hence we find the value of f_{res} in which inequalities are performed:

$$\begin{cases} |-f_{res}| > \dfrac{g^2 - 3h^2e^2\omega^4}{12e\omega^4}; \\ f_{res} > \dfrac{5g^2 + 3h^2e^2\omega^4}{12e\omega^4}. \end{cases}$$

The forces of friction shift boundaries of stability, narrowing the field of instability, which do not reach the axis of abscisses. This means that small ripple parameter ε not to cause friction in the system of parametric oscillation.

The research is just for horizontal rotor systems. There is no parametric oscillation of fluid in vertical rotor systems [3].

4. Movement of fluid taking into account from the angle of the chamber inclination relative to the horizon

If you place the system at an angle β ($0 < \beta < \dfrac{\pi}{2}$) to the horizon line (Fig. 2), the differential equation of the relative motion of the liquid will look like

$$me^2\ddot{\alpha} + (mre\omega^2\cos\theta - mge\sin\beta\cos\omega t)\sin\alpha = 0 . \tag{8}$$

When β=0, we have the case considered above horizontal shaft rotor system, $\beta = \dfrac{\pi}{2}$ - vertical system.

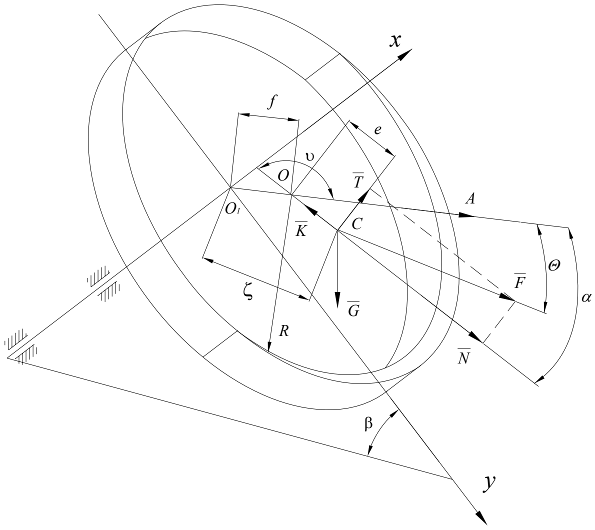

Fig. 2 Fluid in the field of centrifugal forces and gravity at an angle to the horizon

Transform the equation (8) is similar to (1).

We obtain an approximate equation in the form

$$me^2\ddot{\alpha} + me\left(r\omega^2 - g\sin\beta\cos\omega t\right)\alpha = 0\,. \qquad (9)$$

After substituting the equation (9) $r = f + e$, we obtain

$$\ddot{\alpha} + \omega^2\left[\frac{(f+e)}{e} - \frac{g\sin\beta\cos\omega t}{e\omega^2}\right]\alpha = 0 \qquad (10)$$

or

$$\ddot{\alpha} + \omega^2\frac{f+e}{e}\left[1 - \frac{g\sin\beta}{(f+e)\omega^2}\cos\omega t\right]\alpha = 0\,.$$

Turning to the dimensionless time $2\tau = \omega t$, and equation (10) becomes the standard form of Mathieu equation (4), where

$$b = 4\frac{f+e}{e},\ \varepsilon = 2\frac{g\sin\beta}{e\omega^2}\,. \qquad (11)$$

With (11) it is shows that the parameter b depends on the deflection f. The parameter ε is inversely proportional to the angular velocity ω and with its growth decreases.

Condition of liquid stability for selected parameters b, h, ε, β will be written as

$$\begin{cases} b < 4 - \dfrac{\varepsilon^2}{12}; \\ b > 4 + \dfrac{5\varepsilon^2}{12}. \end{cases} \text{or} \begin{cases} 4\dfrac{f+e}{e} < 4 - \dfrac{4g^2\sin^2\beta}{12e^2\omega^4}; \\ 4\dfrac{f+e}{e} > 4 + \dfrac{5\cdot 4g^2\sin^2\beta}{12e^2\omega^4}. \end{cases}$$

Hence we find the value of f_{res} in which inequalities are performed:

$$\begin{cases} \left|- f_{зал}\right| > \dfrac{g^2\sin^2\beta}{12e\omega^4}; \\ f_{зал} > \dfrac{5g^2\sin^2\beta}{12e\omega^4}. \end{cases} \qquad (12)$$

From the relations (12) it is shown that with increasing angle β, residual deflections increase in absolute value. Thus, under the same other conditions of the movement of the system reduce of the angle of inclination to the horizon improves the quality of balancing.

This conclusion confirms the results of experimental studies.

4. Conclusions

1. The movement of fluid in the chamber of ABU for rotors with a horizontal axis of rotation is considered. The influence of damping resistance to fluid movement in the cylindrical chamber is shown.

The forces of friction shift boundaries of stability, narrowing the field of instability, which do not reach the axis of abscisses. This means that small ripple parameter ε not to cause friction in the system of parametric oscillation.

2. Fluid flow with regard to the camera angle of inclination relative to the horizon is analyzed.

Thus, under the same other conditions of the movement of the system reduce of the angle of inclination to the horizon improves the quality of balancing.

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MOVEMENT OF WORKING FLUID IN THE FIELD OF CENTRIFUGAL FORCES AND FORCES OF WEIGHT

S u m m a r y

The article deals with the influence of gravity to work of fluid auto -balancing units (ABUs) of direct action. The movement of fluid in the chamber of ABU for rotors with a horizontal axis of rotation is considered. The influence of damping resistance to fluid movement in the cylindrical chamber is shown. Fluid flow with regard to the camera angle of inclination relative to the horizon is analyzed. To analyze the movements of the working fluid in the field of centrifugal force and gravity forces mathematical tools of parametric oscillation is applied.

Keywords: rotor, vibration, automatic balancing (self-balancing), auto-balancing units (ABU).