Determination of the Characteristics of Viscous Friction in Sliding Bearings

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Abstract: The methods of determining the friction characteristics on the basis of experimental studies using the damped oscillations of the pendulum have been analyzed. It is established that in existing approaches, the results of experimental studies do not always match theoretical models. In this work, we solved the problem of constructing a theoretical model of the pendulum in the sliding bearings with lubricant and on its basis to suggest methods of determining the characteristics of friction: the coefficient of energy absorption by friction and the coefficient of fluid friction - viscosity. It is shown that for a pendulum in lubricated sliding bearings, the process of oscillation is described by the differential equation of the second order with viscous resistance which is proportional to the velocity of deviation of the pendulum. On the basis of the exact solution, it is shown that the ratio of adjacent amplitudes of the damped oscillations is constant, which implies the constancy of the absorption coefficient on the whole process of oscillations. On the basis of the theoretical model of the oscillations of the pendulum it has been obtained that for the viscous friction the coefficient of absorption is exactly equal to twice the logarithmic decrement of damping and is determined by one at a time or a cycle of oscillation. The procedure for determining the dynamic viscosity

Keywords: sliding bearing, lubrication, pendulum, damped oscillations, viscosity

1. Introduction

The material of machine parts has the property to absorb energy during deformation. This physical property of the material, usually called internal friction, is closely connected with its operational properties: strength and durability. Internal friction in the bulk of material have been well studied, and its regularities are widely used in material science when creating materials.
Regularities of the processes of external energy absorption by the surface of machine parts are much less studied. Processes in the surface layer are more complex than in volume. It combines two types of friction: internal and external. These processes occur in very small volumes of material and their study requires the use of rather complex methods of contact mechanics.
Methods for determining the internal friction characteristics are divided into direct: energy, heat, hysteresis loop; and indirect: damped oscillations, resonance curve, phase method.

Test methods and installations that implement these techniques are performed in different configurations: longitudinal or torsional vibrations of tube samples; torsional oscillations of the solid samples; high-frequency longitudinal oscillations of the solid core. The most widely used are testing methods based on the transverse vibrations of samples: when the console fixing a sample; when freely suspended samples, in conditions of pure bending. All the known techniques are implemented on a sufficiently complex installations equipped with devices for automatic measurement of vibration amplitude.
Among the many direct and indirect methods of determining the absorption coefficient the method of damped oscillations is the simplest and physically reasonable. In relation to the study of the energy properties of the surface the method of damped oscillations is the most effectively implemented with using the swinging pendulums.

2. Literature review and problem statement

The method of damped oscillations of a pendulum was first used by Newton for the study of the forces of resistance to the motion of a rigid body in liquids and gases. The obtained results were the basis for the hydrodynamics of laminar flow and modern viscosimetry.
The problem of damping of mechanical vibrations under the effect of viscous resistance of the medium was studied: in the XVIII century: Coulomb (1786); in the XIX century: Laplace (1809), Bessel (1826), Poisson (1831), Stokes (1850), Meyer (1871), Mendeleev D. (1893), who believed that the study of the oscillations of the rocker arm may serve as a sensitive method for the study of gravitational forces and internal friction and planned to use a pendulum for this purpose.
In the twentieth century Akhmatov, A. S. (1939) drew attention to the uniqueness and efficiency of the method of pendulums and have done it for 30 years one of the main methods to study the properties of thin lubricant films in the contact of solids. A wide application of the method of pendulum for the study of friction is explained by the fact that the attenuation is entirely due to external friction in different operating conditions: material; surface condition; temperature; composition of lubricants; the thickness of the layers. It is important that according to the changing the shape of the damping curves it is possible to make conclusions about the influence of these conditions on the friction processes.
A number of papers address theoretical and experimental studies into the methods for determining the energy, absorbed by a surface, including the processes of friction at damped oscillations.
By using the pendulum method, described in monograph [1], it was established that under conditions of viscous friction, oscillation attenuation curve on the elastic layer of molecules has a linear law. However, the oscillation attenuation curve on an unordered layer of lubricant is characterized by the logarithmic law of damping. These data were obtained experimentally and the study lacks a theoretical description of the mechanisms of viscous friction, which explain the results received.
Article [2] examined experimentally and theoretically free fluctuations of a physical pendulum, which rests with two balls on a flat surface. The theory of damping fluctuations proposed in the study for evaluating the energy losses is of a phenomenological nature and is not based on a strict mathematical description of the mechanisms of fluctuation process, which makes it difficult to give the quantitative assessment of losses caused by friction.
In paper [3], a differential equation for the swinging pendulum is obtained, taking into account the deformation of a surface and a deformation component of the friction force. However, the authors examined only a particular case of small deformations, commensurate with the dimensions of contact area of the ball support of a pendulum. This solution is not applicable for describing the macro-displacements in the support of a pendulum, characteristic for the actual friction units of sliding in machines.
Authors of article [4] conducted a nonlinear regression analysis based on the results of experimental studies into the damped oscillations of a physical pendulum, which made it possible to formulate the criteria for choosing a theoretical model of the dissipation of viscous friction. However, the experiments were performed for the conditions of balls rolling on the support, that is, for the small areas of contact in comparison with dimensions of the balls, which also does not make it possible to

adequately utilize the obtained results for evaluating the characteristics of viscous friction in cylindrical sliding supports (bearings).
Paper [5] describes an experimental study of losses to friction in the friction pairs made of metal-polymer materials. In this case, dry friction without the use of a lubricant is examined, which is not characteristic for the cylindrical sliding supports that work under boundary or even liquid friction mode.
Article [6] explores approaches in the development of procedures for determining the characteristics of friction convenient for the practical application under operating conditions of machines. Simple formulas for calculating the coefficient of friction at reciprocating rocking movements on a pendulum device are proposed. The formulas received are not based on the differential equations, which describe the process of viscous friction, and yield only rough estimates of the indices of friction.
Paper [7] deals with the estimated-experimental procedures for determining the characteristics of friction based on a variation approach. This approach implies the use of approximating dependences of experimental data, which renders the solution approximate, depending on the subjective factors.
The experimental method for measuring the characteristics of friction was applied in article [8] for examining special features of high-speed friction under conditions of thermal loading and at limited lubrication. The criterion of damping the friction fluctuations proposed in this case is not applicable to the case of viscous friction, where such fluctuations are practically absent as a result of the damping action of a layer of lubricant.
Paper [9] employed a pendulum method in order to solve a problem on determining the contact rigidity of steel spherical models at dynamic loading using the system of differential equations. However, the tangential component of contact stresses from the frictional forces in this case was not considered.
Articles [10, 11] proposed and implemented design concepts of devices for the accelerated tests on friction and adhesive properties of the actual friction units of the "wheel – rail" type. The design of a pendulum device, applied in the work, for studying the adhesive properties according to the scheme "cylinder – plane" does not allow using it for evaluating the viscous friction of cylindrical sliding supports.

3. The aim and tasks of the study

The aim of present work is the theoretical and experimental study of the characteristics of viscous friction using the method of damped oscillations of the pendulum sliding bearings in with lubricant.
To achieve this aim, the following tasks were to be solved:

- to build a theoretical model of the pendulum in sliding bearings on the basis of differential equations of the second order viscous resistance;
- on the basis of the model of oscillations of a pendulum to suggest methods of determining the characteristics of friction: the coefficient of energy absorption by friction and the coefficient of fluid friction – viscosity;
- to investigate the effect of different types of lubricants and structural materials on the characteristics of viscous friction.

4. The theoretical model of the pendulum in sliding bearings with lubricant for study of viscous friction

Viscosity or internal friction is the most important physical property of oils, which provides separation of solid surfaces and their normal operation with minimal wear.
Tangential force T_{fr} ,arising at the relative sliding of the adjacent fluid layers, is called force of viscous shear (force of fluid friction). This force is determined by Newton's law:

$$T_{fr} = \mu \frac{dV_x}{dy} F \,,$$

where μ - the dynamic viscosity;
F - shear area;
V_x - the speed in the axis direction;
dy - the direction perpendicular to the axis x.

Value:

$$\tau = \frac{T_{fr}}{F} = \mu \frac{dV_x}{dy} \,,$$

is called the strain of viscous shear. In the case of small thickness h of the layer it can be put:

$$\frac{dV_x}{dy} = \text{const} = \frac{V}{h} \,.$$

Then we have: $\tau = \mu \frac{V}{h}$.
At full filling of the gap in the slide bearing with lubrication the operating area of shaft friction on lubrication is equal to:

$$F_{fr} = \pi RB \,.$$

where B is the size of the bearing on forming.
The shear force along the entire circumference:

$$T_{fr} = \tau F_{fr} = \mu \frac{V}{h} 2\pi RB \,.$$

If we consider that the linear sliding velocity is equal to:

$$V = R\dot{\varphi} = R\frac{d\varphi}{dt} \,,$$

we have:

$$T_{Tp} = \frac{\mu}{h} 2\pi R^2 B\dot{\varphi} \,. \tag{1}$$

The moment of resistance to rotation of the shaft from the viscous lubricant:

$$M_{\mu} = \mu^* R^2 \dot{\varphi} \,. \tag{2}$$

where:

$$\mu^* = \frac{\mu}{h} 2\pi RB \,. \tag{3}$$

Except for viscous resistance to rotation onto the shaft there are two other points: the moment from inertial force:

$$M_j = J\ddot{\varphi} = ml^2 \ddot{\varphi} \,. \tag{4}$$

and the moment from the weight of the pendulum:

$$M_Q = Ql\sin\varphi = mgl\sin\varphi\,. \tag{5}$$

The equilibrium equation includes the total of all points:

$$M_\mu + M_j + M_Q = 0\,. \tag{6}$$

or

$$ml^2\ddot{\varphi} + \mu^* R^2\dot{\varphi} + mgl\sin\varphi = 0\,. \tag{7}$$

At small oscillations $\varphi \approx \sin\varphi$ we have the classical equation of oscillations with viscous resistance:

$$\ddot{\varphi} + 2n\dot{\varphi} + p^2\varphi = 0\,. \tag{8}$$

where

$$2n = \frac{\mu^* R^2}{ml^2}\,.$$

Taking into account (3):

$$n = \frac{\pi\mu R^3 B}{hml^2}; \quad p^2 = \frac{g}{l}; \tag{9}$$

The characteristic equation of the differential equation of oscillations (8):

$$s^2 + 2ns + p^2 = 0$$

has two imaginary roots:

$$s = -n \pm i\sqrt{p^2 - n^2} = -n \pm ip_1\,; \; p_1 = \left(p^2 - n^2\right)^{\frac{1}{2}}.$$

Having the linear roots of the equation (8) they have the form:

$$\varphi = e^{-nt}\left(c_1\cos p_1 t + c_2\sin p_1 t\right) \tag{10}$$

or, denoting:

$$c_1 = A\cos\varphi\,; \quad c_2 = A\sin\varphi\,.$$

we have a different form of the solution:

$$\varphi = Ae^{-nt}\cos(p_1 t + \varphi) \tag{11}$$

where

$$A = \left(c_1 + c_2\right)^{\frac{1}{2}}; \quad \mathrm{tg}\varphi = -\frac{c_2}{c_1}\,. \tag{12}$$

The constants c_1 and c_2 are determined from initial conditions:

$$\varphi(t=0) = \varphi_0; \dot{\varphi}(t=0) = 0 \tag{13}$$

Taking into account the solution (12), we have:

$$c_1 = \varphi_0; c_2 = -\varphi_0 \frac{n}{p_1}. \tag{14}$$

Substituting the constants in (12), we have:

$$A = \varphi_0 \left(1 + \frac{n^2}{p_1^2}\right)^{\frac{1}{2}}, \tag{15}$$

From the solution it follows that that if there is viscous friction the motion of the payload is described by periodic damped oscillations with a period:

$$T = \frac{2\pi}{p_1} = \frac{2\pi}{\sqrt{p^2 - n^2}}, \tag{16}$$

where p_1 - angular frequency of damped oscillations.
From the ratio of two successive maximum deviations of the pendulum from the equilibrium position

we have:

$$\varphi_k = \varphi_{k+1} e^{nT}, \tag{17}$$

which corresponds to a geometric progression.
In practice, they often use the logarithm from the value of the ratio of adjacent amplitudes, which is called logarithmic decrement of damping:

$$\delta = \ln \frac{\varphi_k}{\varphi_{k+1}} = nT \tag{18}$$

If the oscillations of damping are slow, and the ratio $\frac{\varphi_k}{\varphi_{k+1}} \approx 1$ is close to one, then:

$$\delta = \ln \frac{\varphi_k}{\varphi_{k+1}} = \ln \frac{\varphi + \Delta \frac{\varphi}{2}}{\varphi - \Delta \frac{\varphi}{2}} \approx \frac{\Delta\varphi}{\varphi} \tag{19}$$

where

$$\Delta\varphi = \varphi_k - \varphi_{k+1}; \varphi = \frac{(\varphi_k + \varphi_{k+1})}{2}.$$

Thus, at slow damping the logarithmic decrement is approximately equal to the ratio of the amplitude changes for the period T to the average amplitude φ.

5. The results of determination the characteristics of viscous friction in sliding bearings by the pendulum method

As a result of the above theory of oscillations of the pendulum in the sliding bearing with lubrication the methodology for determining the friction characteristics has been proposed: the absorption

coefficient of energy in friction and fluid friction coefficient - viscosity. The pendulum is the perfect instrument for determining the energy absorbed by the surface in contact. The reduction in amplitude per cycle is connected with a decrease of potential energy of the load, causing fluctuations. The smaller the losses are, the longer the pendulum oscillates, and the longer the oscillations are, the more accurately we can measure the energy losses. The main regularity of the oscillations of a pendulum with viscous friction has the form type (17).
By definition the absorption coefficient ψ is called the quantity equal to the ratio of energy ΔP lost per cycle to the total energy P per cycle. Taking into account (19) we have:

$$\psi = \frac{\Delta P}{P} = \frac{2\Delta\varphi}{\varphi} \tag{20}$$

Thus:

$$\psi = 2\frac{\varphi_k - \varphi_{k+1}}{\varphi_k} = \frac{2\Delta\varphi}{\varphi} = 2\ln\frac{\varphi_k}{\varphi_{k+1}} \tag{21}$$

or

$$\psi = 2\left(1 - \frac{\varphi_{k+1}}{\varphi_k}\right) = 2\left(1 - e^{nT}\right) \tag{22}$$

For calculations of the absorption coefficient according to the test results in the viscous friction we can consider a few limits and approximately determine the averaged value of the coefficient according to the formula by Sorokin E. S.:

$$\psi = \frac{2}{N}\frac{\varphi_k - \varphi_{k+N}}{\varphi_k} \tag{23}$$

The procedure for determining the absorption coefficient for damped oscillations of pendulum is the following: the envelope curve $\varphi(t)$ is built; for two adjacent amplitudes the coefficient of absorption according to the formula (22) is defined; or to the site on the envelope from φ_k to φ_{k+N} the calculation of the absorption coefficient by the formula (23) is carried out.
An example of determining the coefficient of absorption of energy by friction. It is necessary to determine the absorption coefficient of energy in the contact of the sliding bearing with lubricant under the following initial conditions on the pendulum: $l = 350$ mm; $Q = 2$ N; $R = 15$ mm; $B = 20$ mm; $h = 0,05$ mm. From the experiment on the envelope is set: $T = 1$ c, $\frac{\varphi_k}{\varphi_{k+1}} = 1,19$. According to the formula (24) we find: $n = \frac{1}{T}\ln\frac{\varphi_k}{\varphi_{k+1}} = \ln 1,19 = 0,1739c^{-1}$. The absorption coefficient is determined by the formula: $\psi = 2\ln\frac{\varphi_k}{\varphi_{k+1}} = 0,3479$.
On the basis of the obtained results let us also consider the method of determining the cofficient fluid friction (viscosity).
If from the experiment the justice of dependence on the type (17) has been set and the period of oscillations T has been found, then from (18) we can find:

$$n = \frac{1}{T}\ln\frac{\varphi_k}{\varphi_{k+1}} \tag{24}$$

From (9), we have ($g = 9810\frac{mm}{s^2}$):

$$n = \frac{\pi\mu R^3 B}{hml^2} = \frac{\pi\mu R^3 Bg}{hQl^2} \quad (25)$$

The value of dynamic viscosity is determined from (25):

$$\mu = \frac{nhQl^2}{\pi R^3 Bg} \quad (26)$$

At a small load the oil film thickness can be taken equal to the gap: $\mu = \Delta = R_2 - R_1$, where R_2 is the radius of the bushing; R_1 -radius of the shaft.

An example of determining the dynamic viscosity of lubricants. The tests were carried out on a specially designed pendulum device. The parameters of the pendulum device are as following: the length of the pendulum $l = 350$ mm; the weight of the pendulum 2…6 N; the radius of the bearing, $R = 15$ mm; bearing width $B = 20$ mm; the thickness of the lubricant layer (equal to the gap) $h = 0,05$ mm.

During the test, four types of support materials and three types of lubricants were used (Table. 1).

Table 1. Test results

Bearing material		Load								
		Motor Oil SAE 15W-5			Formula Q8			Solidolide G2		
		2N	4N	6N	2N	4N	6N	2N	4N	6N
Plexiglass	N	22	26	28	18	22	26	7	11	16
	n	0,24	0,19	0,18	0,29	0,24	0,19	0,82	0,49	0,33
	μ	2,83	4,48	6,47	3,43	5,65	6,71	9,66	11,6	11,6
Copper	N	13	15	18	10	13	16	5	9	14
	n	0,41	0,35	0,29	0,55	0,41	0,33	1,22	0,61	0,38
	μ	4,85	8,32	10,3	6,47	9,71	11,6	14,4	14,4	13,4
Brass	N	18	24	28	14	19	22	6	12	16
	n	0,29	0,21	0,18	0,38	0,27	0,24	0,99	0,45	0,33
	μ	3,43	5,06	6,47	4,48	6,48	8,48	11,7	10,6	11,6
Bronze	N	14	17	19	12	14	17	5	9	13
	n	0,38	0,31	0,27	0,45	0,38	0,31	1,22	0,61	0,41
	μ	4,47	7,28	9,72	5,29	8,95	10,9	14,4	14,4	14,6

Note: $\mu -$ the number of oscillations to the standstill of pendulum; $n -$ coefficient of viscous resistance of lubricant; $\mu -$ dynamic lubricant viscosity, $Pa \cdot s$.

The pendulum was deflected by the specified angle (90^0) and recorded the decrease in the amplitude of the oscillations until the pendulum was completely stopped. The analysis of the results shows that for all combinations of structural materials under lubrication of liquid oils with increasing load there is an increase in viscosity. When greasing with lubrication the viscosity characteristics are not substantially changed. They have a higher load capacity.

The obtained values of the viscosity characteristics of liquid lubricants differ for different combinations of structural materials. The analysis obtained results shows that the viscosity varies from the smallest to largest values for the tested oils in the following order: plexiglass – brass – bronze – copper.

This is due to the different strength of adhesion of lubricants and surfaces of the materials tested. The degree of adhesion can be indirectly evaluated by the contact angle of wetting the surface of the materials. The better the wetting is, the stronger the adhesion can be.

The contact angle of wetting Θ was measured in the following way. A drop of the test liquid was pipetted onto the surface, illuminated with a strong light source (a point source is recommended), and projected onto the screen. The hard surface was placed parallel to the beam of rays, so that the

projection of the hard surface was projected onto the screen in the form of a thin line. The image of the drop on the screen was delineated or photographed.
At the point of intersection of the projection of the drop and surface the tangent was built and the value of the contact angle was determined. It is desirable to have an enlarged image of the drop silhouette, which can be obtained with the help of a projector.
As an example, the angle of wetting the surface of samples from plexiglass, brass, bronze and copper with motor oil SAE 15W-5 was measured. The results of the wetting angle measurements are presented in Table 2.

Table 2.The results of the wetting angle measurements

Material	Plexiglass	Brass	Bronze	Copper
Contact angle of wetting Θ , deg	41	29	23	19
$\cos\Theta$	0,755	0,875	0,921	0,945

From the analysis of the results of the wetting angle measurements we have obtained that the best wettability and the highest adhesion has the contact oil of copper. The wetting angle decreases further in the order: bronze, brass, plexiglas. The obtained data correlate well with the results of determination of viscosity on the pendulum device (see table. 1).

6. Discussion of results of determination the characteristics of viscous friction in sliding bearings

A stronger adhesion determines a stronger grip of the parietal layer of the lubricant with the surface of the material. This determines the high rates of viscous resistance at a relative movement of the liquid layers between solid surfaces. The obtained results allow to hold certain discussions. In accordance with the concept of Newton there is a full-grip of surfaces in the contact of solids and liquid. Thus, the velocity of the points of the surface of the liquid and the points of the surface of the solid body coincide. In this case, the solid moving through the liquid surface, must drag along all points of the surface with which it comes into contact.
In fact, the contact points of the liquid surface are in contact with the solid only at some limited area, and then are out of the contact and slip off the surface. Therefore, the surface of the liquid slides on the surface of solid bodies and between solid and liquid surface and there is an external friction. It is not considered in classical hydrodynamics.
Let us denote the limiting tangential forces between the layers of fluid τ_f and the tangential forces between solid and liquid τ_s. Then two cases are possible

1) $\tau_f < \tau_s$ – sliding occurs between the fluid layers – this is internal friction in fluid;
2) $\tau_f > \tau_s$ – sliding occurs between the solid and the liquid surface – this is external friction.

It is obvious that they are different types of resistance to movement of body in fluid. In the general case of motion of a solid body over a liquid, these two types of resistance are combined and it is not easy to separate them. At the same time, such a separation is possible by conducting special experiments.
In some methods to determine the viscosity of the whole volume the deformation of the lubricant occurs. In the determination of the kinematic viscosity the viscometers of variable cross section are used. When the lubricant flows through such a viscometer, it experiences volumetric deformations during the transition to the narrow cross-section and shear deformations when passing through narrow places. In this case the number of viscosity consists of two components: the volumetric deformations and shear deformations. The solution to the problem of separation of these components seems to be promising for the theory of lubrication.
Thus, it is possible to believe that the phenomenon of viscosity consists of three components: internal friction from shearing between the layers of the lubricant; external friction of solid bodies and

lubricants; internal friction and resistance of volumetric deformation of the lubricant. Lubricant has both properties of a solid deformable body (skeleton) and liquid (oil). Under the current review of the mechanical properties of lubricant, particularly in thin layers, stiffness characteristics are practically not taken into account. This limits the solution of problems of contact interaction of bodies separated by a layer of lubricant. The study of normal and tangential stiffness properties of thin films in the contact between deformable bodies is one of the basic problems of tribology of oil films.

7. Conclusions

- The main advantage of the method of pendulums in determining the tribological characteristics consists in increasing the accuracy of measurements with increasing duration of oscillations. The first, who drew attention to the effectiveness of the pendulum method, was the great Isaac Newton, who with the help of this method discovered the law of motion of solid bodies in liquids. In recent times the pendulum method attracts the attention of researchers again. The method is improved in the direction of improving the accuracy and broadening applications.
- In the case of the pendulum lubricated sliding bearings the process of oscillations is described by the second order differential equation with viscous resistance proportional to the velocity of the pendulum deflection. The analysis of the exact solution of this problem shows that the ratio of the adjacent amplitudes of damped oscillations is constant. Hence this is followed by the constancy of the absorption coefficient during the whole process of oscillations.
- The ability of a surface to absorb the energy of the contact deformation is one of the most important physical and mechanical properties of the solid. It is convenient to perform the determination of the absorbed energy using the method of pendulums. For the case of viscous friction, the absorption coefficient is exactly equal to twice the logarithmic decrement of damping and is determined by either one or the cycle of vibration. An important result for the practice is the ability to determine the dynamic viscosity of the lubricant in contact according to the decrement of pendulum damping oscillations.

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