Taras Shevchenko National University of Kyiv

Institute of Mathematics of NAS of Ukraine

Ukraine Algebra Conference "At the End of the Year 2023"

December 26 – 27, 2023

Kyiv, Ukraine

BOOK OF ABSTRACTS

Kyiv — 2023

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C O N T E N T S

3D QUASI-MONOMIALS

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Let H be a subgroup of the space affine group Aff(3) considered with the natural action on the vector space of three-variable polynomials. The polynomial family $B_{m,n,k}(x,y,z)$ is called *quasi-monomial* with respect to H if the group operators in two different bases $\{x^m y^n z^k\}$ and $\{B_{m,n,k}(x,y,z)\}$ have *identical* matrices. Quasi-monomials are widely used in the analysis of 2D and 3D images, see [1].

In [1], a description of all quasi-monomial families with respect to the plane rotation group $SO(2)$ in terms of their generating functions is provided. Additionally, in [2], a description of quasi-monomials with respect to groups of scaling and translation of the plane was obtained.

The quasi-monomial family $\{B_{m,n,k}(x,y,z)\}_{m,n=0}^{\infty}$ is called quasi-monomial family with respect to the translation group if the following identity holds

$$B_{m,n,k}(x+a,y+b,z+c)=\sum_{s=0}^{m}\sum_{p=0}^{n}\sum_{t=0}^{k}\binom{m}{s}\binom{n}{p}\binom{k}{k}a^{m-s}b^{n-p}c^{k-t}B_{s,p,t}(x,y,z)$$

for all $m,n,k\in\mathbb{N}$.

We propose a criterion for quasi-monomiality when the group H is the space translation group, using the exponential generating function associated with the polynomial family $B_{m,n,k}(x,y,z)$.

Theorem. *The polynomial family $\{B_{m,n,k}(x,y,z)\}$ defined by the exponential generating function*

$$G(x,y,z)=\sum_{m,n,k=0}^{\infty}B_{m,n,k}(x,y,z)\frac{u^m}{m!}\frac{v^n}{n!}\frac{w^k}{k!}$$

is quasi-polynomial with respect to the space translation group if and only if its exponential generating function has a form

$$G=C(u,v,w)e^{xu+yv+zw},$$

where $C(u,v,w)$ is an arbitrary power series in variables u,v,w.

1. Bedratyuk L., Flusser J., Suk T., Kostkova J., Kautsky J. Non-separable rotation moment invariants. Pattern Recognition, 2022, 127, 108607.
2. Samaruk N. Quasi-monomials with respect to subgroups of the plane affine group. Mat. Stud., 2023, 59(1), 3–11.

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