A Novel Method for Determining Weighting Coefficients in the Block Convolution Model of the Managerial Decisions Effectiveness Comprehensive Assessment

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***Abstract*—The development of effective managerial decisions is the most crucial stage in management activities, a necessary condition for ensuring the competitiveness of a business entity. The complexity and high dynamism of the market environment make it essential to consider a large number of partial criteria when choosing the most effective alternative. The analysis of modern studies showed that despite many different approaches to evaluating alternatives, they are mainly based on aggregating initial criteria. Such a procedure can be carried out by applying the technology of comprehensive index assessment. A large number of partial criteria complicates the procedure for constructing a comprehensive index, makes it cumbersome, reduces its informativeness and discriminating ability, and negatively affects the significance of the weighting coefficients. To get rid of these shortcomings, the article considers the procedure of block convolution, in which the initial criteria are first divided into groups according to specific rules. A partial comprehensive index (criterion) is constructed for each group. At the next stage, they are integrated into a comprehensive index of the effectiveness of alternatives by weighted convolution. For the case of two partial comprehensive indexes, a method of dynamic weighting coefficients has been developed to evaluate each component's importance in their convolution. Its feature is the consideration of a more significant partial comprehensive index obtained at the first stage of convolution, which reduces the compensatory influence of the second partial comprehensive index.**

Keywords—block convolution, decision-making, multicriteria, dynamic weighting coefficients method.

I. Introduction

To develop managerial decisions, it is essential to correctly assess the current situation and alternative decisions with appropriately determining the signs of a practical decision that meets the goals of the business entity. A correct assessment contributes to achieving the set goals, while an erroneous evaluation and, as a result, an incorrect decision make it difficult to achieve the desired result. The multi-criteria description of alternative managerial decisions and the potential inconsistency of evaluations of their effectiveness based on different criteria lead to the solution of the non-trivial task of finding an acceptable alternative as a managerial decision. The objective multidimensionality of the source data, caused by the complexity of market processes and phenomena, requires information compression methods to present it in a compact and comprehensible form. The solution to such tasks is possible by using the data aggregation toolkit in compliance with the requirements of preserving their informativeness. Aggregation aims to obtain generalized characteristics of the studied processes, although the prerequisites of tasks related to this purpose may differ. Therefore, solving issues related to evaluating the effectiveness of alternative options of managerial decisions according to the selected set of criteria is a relevant issue.

II. Literature Review

Published research results analysis in recent years shows that studying issues related to the alternatives' effectiveness evaluation in conditions of multicriteria remains the focus of research. At the same time, most approaches are aimed at aggregating raw data. One of the most widely used approaches is applying the TOPSIS method, which consists of comparing alternatives by evaluating the distances in the multidimensional space of their attributes to the best and worst alternatives. Particularly, such an approach in economics and management fields was reflected in the evaluation of management decisions in tourism [1] and the selection of suppliers for manufacturing and service companies [2]. Studies [3], [4], [5], [6], [7] proposed to improve this method, in particular, by choosing other metrics for evaluating the distance to the "ideal" decision and selecting several reference points for assessing the effectiveness of alternatives. Sunil [8], Yang with co-authors [9], and Ardil [10] consider the application of fuzzy sets theory for managerial decision-making. In the article [11], Zhang and Dai combined this theory and the TOPSIS method. The authors considered the distances to the best and worst alternatives in terms of fuzzy sets. This made it possible to improve the technique of classifying alternatives both for the case of their belonging to joint decision area and for belonging to different decision areas. Studies [12], [13], [14], [15] present the results of applying the gray target decision method to the problem of choosing the most suitable alternative as a decision. In particular, the authors propose new approaches to normalizing initial data and determining weighting coefficients of attributes when evaluating the weighted distance to the "ideal" point. It should be noted that the presented algorithms require rather complex calculations, which limits their practical application. An overview of other approaches to solving Multi-Criteria Decision Making problems is given in studies [16], [17]. The analysis of the cited sources shows that, despite many different approaches, they are based on the idea of building some comprehensive measure that reflects the degree of effectiveness of each of the alternatives. At the same time, one of the critical problems remains the determination of the weighting factors of the components during convolution.

Our research aims to develop a method for determining weighting coefficients in the block convolution of partial generalized criteria when evaluating the effectiveness of alternative managerial decisions.

III. Problem Description and Methodology

Let us consider a specific system that contains a set of alternatives and certain of their characteristics, which reflect the properties of these alternatives concerning the a priori idea of their effectiveness:

$$S = \langle A^{(0)}, \Delta \rangle, \quad (1)$$

where $A^{(0)}$ $^{)}$=$\{A_1, A_2, \ldots, A_m\}$ is a set of alternatives (further on, it will also be interpreted as a set of objects that will be scaled within the framework of a comprehensive assessment of their latent quality);

Δ is the structure of the system, i.e., some pair relation defined on a set of pairs of alternatives $A^{(0)} \times A^{(0)}$ that reflects the characteristics of the relationships between them;

m – number of alternatives.

A square matrix can always represent such a relationship Δ, the elements of which δ_{ij} reflect the relationships between objects, i, j =1, 2, …, m. They are usually measured on the metric scale. In particular, the role of such relations can be the distance between objects in a multidimensional feature space (Euclidean, Hamming, and others), connection coefficients (correlations, associativity, information measures, etc.), results of expert evaluation of objects, preference relationships, and others. At the same time, using indicators of correlation allows us to picture not only the degree of this relationship but also its direction and intensity, while other measures evaluate only its quantitative characteristics.

Some set of criteria usually characterizes alternatives. Hence, the relationship between them depends on the sense in which the system's structure is considered and may also differ by the nature of the elements and the method of obtaining them. The most used is the use of preference relationships and correlations. In the latter case, the criteria are interpreted as indicators that reflect the decision's effectiveness.

The matrix Δ can be formed in different ways. If it is evaluated based on those indicators that characterize the set of alternatives, that is, the internal information of system S, then it is internal. If the elements δ_{ij}, i, j =1, 2, …, m. are determined based on some characteristic external to the system S, for example, indicators that are not included in the aggregate based on which the construction of alternatives is carried out, but reflect the consequences of their implementation, and then the relation Δ is also external. At the same time, it is necessary to establish a connection between the internal structure of system S (the manifestation of which is set based on external information) and the system of internal indicators.

We evaluate the alternative's effectiveness based on a comprehensive index. The initial data are partial criteria reflecting evaluative judgments about the studied objects (alternatives). It is usually assumed that the generalized criterion for the direction of a positive change in values is their growth. In the presence of many partial criteria, a contradiction arises in determining the superiority of some alternatives over others and in obtaining a generalized assessment of their effectiveness. This necessitates the convolution of partial criteria. The most common methods of solving this problem are additive, multiplicative convolution, and the technique of distances. At the same time, the partial criteria are normalized, making it possible to obtain a generalized criterion in a normalized form. This form is more convenient for interpreting the results, particularly for comparing the evaluation results by different convolution methods.

IV. Results and Discussions

The set of criteria selected for evaluating alternatives should be complete, i.e., reflect all essential characteristics of alternatives; available for obtaining evaluations from all received criteria; devoid of elements of duplication in the assessment of decision options; to have a sufficient resolution concerning alternatives, concise, which will save the multi-criteria evaluation procedure from cumbersomeness and complexity. The presented requirements are quite contradictory, so their integral convolution in the form of a comprehensive index is an effective means of solving this problem.

The set of indicators that be included as partial criteria to construct the comprehensive index is presented in the form:

$$X == \{X_1, X_2, \ldots, X_n\}, \quad (2)$$

where X_i is an i-th indicator (criterion), n – number of indicators.

Indicators are normalized with compliance with the increasing value of their best quality. This procedure is necessary so that the constructed comprehensive index is within [0; 1] and positively correlates with each component. Normalization can be carried out according to [18], depending on the type of criterion. We denote the normalized criteria by $\widetilde{X}_j^{(i)}$, j=1..n. Based on the normalized values of the indicators, we calculate the correlation matrix R. Its values act as the matrix Δ.

A large number of partial criteria complicates the procedure for constructing an integral indicator, makes it cumbersome, reduces its informativeness and discriminating ability, and negatively affects the significance of the weighting coefficients. A way out of the situation can be a block convolution procedure, in which a specific characteristic first groups the initial criteria. At the same time, the following conditions are proposed for each group:

1) Indicators of each group should reflect one characteristic of the objects under study.

2) There should be fairly close correlations between the indicators within the group.

3) Correlations between indicators of different groups should be insignificant.

The initial set of partial criteria can be divided into blocks according to their content load. For example, it is possible to separately distinguish a group of partial criteria for the efficiency of resource provision, a group of criteria for the effectiveness of pricing, a group of criteria characterizing the financial condition of a business entity, etc. Grouping can be done using dimensionality reduction methods. In particular, you can use computational algorithms of the correlation pleiades method, the method of potential, etc.

This process can occur in several stages until an acceptable number of baseline criteria is reached. At the same time, it should be noted that in the following steps, the first and second conditions, generally speaking, will not be fulfilled. The disadvantage of this approach is the loss of a direct connection between the initial indicators and the final integral indicator, which complicates its use concerning the generalized effectiveness evaluation for new alternatives.

A partial comprehensive index is determined for each group:

$$F_i = \sum_{j=1}^{k_{1i}} w_j^{(i)} \tilde{X}_j^{(i)}, \quad (3)$$

where i – group number, i=1, 2, ..., p;

k_{1i} – number of partial criteria are included in the i-th group;

F_i — is a partial comprehensive index calculated for the i-th group;

$\tilde{X}_j^{(i)}, w_j^{(i)}$ – are base indicators included in the i-th group and appropriate weighting coefficients, j=1, 2, …, k_{1i}.

A significant problem is identifying weighting coefficients $w_j^{(i)}$. A possible solution is the aggregation of partial criteria by the principal components method. In this case, partial comprehensive index F_i can be obtained without directly applying formula (3). At the same time, the division of the original set of criteria into homogeneous groups is also indirectly obtained. Their number is determined by the selected eigenvalues of the corresponding correlation matrix, and the filling is determined by the most significant factor loadings of the related main components [19]. We take this approach as a basis for further considerations.

At the next stage, the weighted Euclidean distance d_i, i=1..n, is calculated in the space of partial comprehensive indexes from the initial objects to the standard, which has coordinate values equal to 1:

$$d_i = \sqrt{\sum_{j=1}^{p} w_j \left(F_{ij} - 1\right)^2}, \quad (4)$$

where F_{ij} is the value of the j-th partial comprehensive index for the i-th alternative, i=1, 2, ..., m. Taking into account the application of the principal components method at the previous stage, the weighting coefficients w_j are calculated according to the formula:

$$w_j = \frac{\lambda_j}{\sum_{s=1}^{p} \lambda_s}, \quad (5)$$

where j=1, 2, …, p,

p – number of groups.

The formula determines the value of the generalized comprehensive index:

$$I_i = 1 - d_i \quad (6)$$

For the case p = 2, when two partial comprehensive indexes F_1 and F_2, were formed at the first step of the block convolution, we propose a new procedure for their aggregation, which was named the method of dynamic weighting coefficients. Its essence consists in the use of additive convolution, for which the weighting coefficients are not constant, but are calculated in proportion to the distances from the point P_i, the coordinates of which are the values of the partial indicators, i=1, 2, ..., m, to the lower and upper poles of the space of permissible values of these indicators with coordinates $P_H(0, 0)$ and $P_B(1/\sqrt{2};1/\sqrt{2})$:

$$I = \frac{d_H F_{i1} + (1 - d_B) F_{i2}}{\sqrt{2}}, \quad (7)$$

where $d_H = \sqrt{F_{i1}^2 + F_{i2}^2}$ – is the distance from the point (F_{i1}, F_{i2}) to the lower pole;

$d_B = \sqrt{\left(\frac{1}{\sqrt{2}} - F_{i1}\right)^2 + \left(\frac{1}{\sqrt{2}} - F_{i2}\right)^2}$ – is a distance from the point (F_{i1}, F_{i2}) to upper pole; i = 1, 2, …, m.

The use of expression (7) involves additional normalization of the values of the partial comprehensive indexes F_1 and F_2 so that their values belong to the segment [0, $\frac{1}{\sqrt{2}}$]. The multiplier $\frac{1}{\sqrt{2}}$ is chosen from the requirement that the values of the indicator I of the segment [0; 1]. For the same purpose, the initial values of the partial integral indicators must first be reduced to the segment [0; $\frac{1}{\sqrt{2}}$]. The weighting coefficients in expression (7) are chosen in such a way that the partial integral indicator F_1, which corresponds to the first principal component (and, accordingly, a larger eigenvalue of the correlation matrix R, and therefore has a higher informativeness), should have a greater weight in the convolution.

Let us consider some properties of the proposed convolution method. Let's calculate the value of the comprehensive index at the extreme points.

When F_{1i} = 0, F_{2i} =0, the value of $I^{(H)}$ = 0 (for the lower pole, there is a zero indicator value).

When $F_{1i} = \frac{1}{\sqrt{2}}$, $F_{2i} = \frac{1}{\sqrt{2}}$, value $I^{(B)} = 1$. Indeed, in this case: $\begin{cases} d_H = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1; \\ d_B = 0; \end{cases}$. Then

$$I^{(B)} = \frac{1 \cdot \frac{1}{\sqrt{2}} + (1-0) \cdot \frac{1}{\sqrt{2}}}{\sqrt{2}} = 1.$$

Let us prove that the index I calculated according to formula (7) corresponds to the ratio of the superiority of the first partial comprehensive index F_1 over the second index F_2: its value at some point $P_M(x, y)$, $x>y$ is greater than the value at the point $P_N(y, x)$. Note that in this case, the distances from the points P_M та P_N to poles P_H та P_B are the same (Fig. 1), which follows, in particular, from the isosceles triangles $P_H P_M P_N$ and $P_B P_M P_N$.

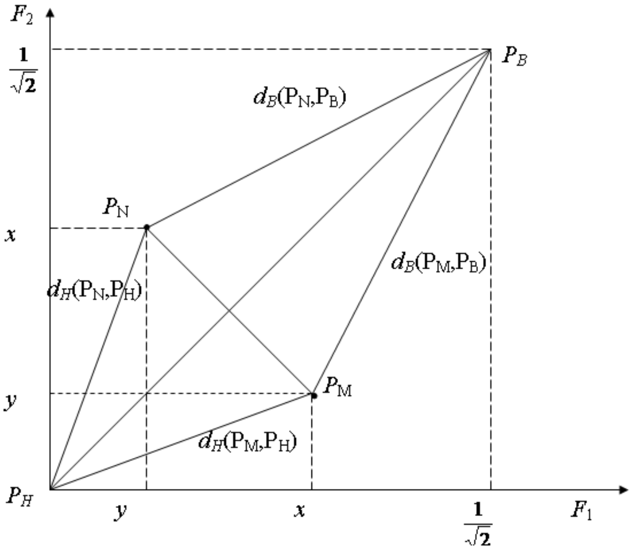

Fig. 1. A picture of points P_M and P_N and corresponding distances

Let's enter the notation:

$$\begin{cases} d_H(P_M, P_H) = d_H(P_N, P_H) = d_{H0}; \\ d_B(P_M, P_B) = d_B(P_N, P_B) = d_{B0}. \end{cases} \quad (8)$$

Let's find the difference in comprehensive index values in points P_M and P_N:

$$I^{(M)} - I^{(N)} == \frac{(d_{H0} + d_{B0} - 1)(x - y)}{\sqrt{2}}. \quad (9)$$

Taking into account that $x > y$ $d_{H0} + d_{B0} \geq 1$, we get:

$$(d_{H0} + d_{B0} - 1)(x - y) \geq 0, \quad (10)$$

or:

$$I^{(M)} \geq I^{(N)}, \quad (11)$$

which had to be proved.

For the point $P_O(x, x)$, we get the following values of the distances to the upper and lower poles:

$$\begin{cases} d_H(P_O, P_H) = x\sqrt{2}; \\ d_B(P_O, P_B) = 1 - x\sqrt{2}. \end{cases} \quad (11)$$

Substituting (11) into (7), we get the value of the comprehensive index:

$$I^{(O)} = \frac{x\sqrt{2} \cdot x + (1 - (1 - x\sqrt{2}) \cdot x}{\sqrt{2}} = 2x^2. \quad (12)$$

Expression (12) can be used to calculate the values of the comprehensive index under the given conditions instead of the formula (7).

Let us prove that the increase in the values of comprehensive index I correspond to the directions of positive changes in its components. Consider two points $P_{M1}(x_1, y)$ and $P_{M2}(x_2, y)$, $x_2>x_1$, in which the value of the coordinate corresponding to F_2 is constant. We denote by d_{H1}, d_{B1}, d_{H2}, d_{B2}, respectively, the distances from these points to the lower and upper poles. Let's find the difference in the values of the integral indicator at these points:

$$I^{(M2)} - I^{(M1)} = \frac{d_{H2}x_2 - d_{H1}x_1 + (d_{B1} - d_{B2})y}{\sqrt{2}} \quad (13)$$

$d_{H2} > d_{H1}$, $d_{B1} > d_{B2}$ (Fig. 2). Therefore, the value of (13) also be greater than zero, which means that a more significant value of the partial comprehensive index F_1 with equal values of F_2 corresponds to a more substantial value of the generalized comprehensive index I. Similarly, this statement is proved for the second partial comprehensive index F_2.

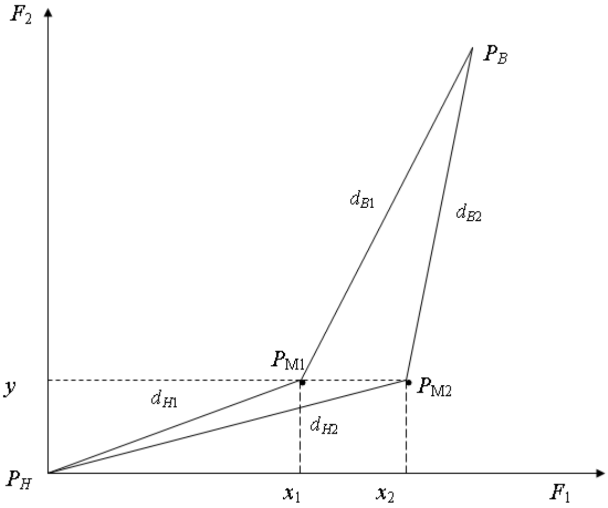

Fig. 2. Graphical interpretation of the direction of positive changes in the partial comprehensive index F_1

In this case, the modified comprehensive index takes the form:

$$I_M = \sqrt{2}(w_1 F_1 + w_2 F_2). \quad (14)$$

The comprehensive index calculated in this way has one drawback - the above methodological approach is not suitable for its interpretation since the weighting coefficients in the sum are different from 1. Therefore, it is advisable to carry out normalization of the weighting coefficients:

$$\begin{cases} w_1 = \dfrac{d_H}{d_H + 1 - d_B}; \\ w_2 = \dfrac{1 - d_B}{d_H + 1 - d_B} \end{cases}. \quad (15)$$

Let us interpret the results of evaluating the effectiveness of alternatives based on the constructed comprehensive index from the view of their acceptability. To meet this aim, we use Harrington's desirability scale [20]. The correspondence between the preference relations in the empirical and numerical systems is shown in Table. I.

TABLE I. THE RELATIONSHIP BETWEEN THE QUANTITATIVE VALUES OF THE DIMENSIONLESS DESIRABILITY SCALE AND THE DESIRABILITY LINGUISTIC EXPRESSION

Desirability	Marks on the desirability scale
Very bad	0.00..0.20
Badly	0.20..0.37
Satisfactorily	0.37..0.63
Good	0.63..0.80
Very good	0.80..1.00

Let's illustrate the presented method on a conditional example of evaluating the effectiveness of eight alternatives according to five metric criteria. The initial data are shown in the Table. II.

TABLE II. NORMALIZED VALUES FOR THE EIGHT ALTERNATIVES MEASURED BY FIVE METRIC CRITERIA

Alternatives	Criteria				
	Q_1	Q_2	Q_3	Q_4	Q_5
A_1	0.00	0.45	0.00	0.16	0.11
A_2	0.43	0.42	0.17	0.00	0.25
A_3	0.43	0.47	0.67	0.08	0.50
A_4	1.00	0.00	1.00	0.71	0.00
A_5	0.46	0.69	0.40	0.66	0.63
A_6	0.86	1.00	0.20	1.00	0.50
A_7	0.46	0.36	0.42	0.90	1.00
A_8	0.64	0.69	0.67	0.80	0.63

The first two eigenvalues of the corresponding correlation matrix are λ_1=2.09 and λ_2=1.81. The total level of informativeness of two partial comprehensive indexes is: $(\lambda_1 + \lambda_2) / \sum_{j=1}^{5} \lambda_j \approx 0.78$, which corresponds to an acceptable level of informativeness (at least 75%.

Let us convolute the partial efficiency criteria by the method of dynamic weighting coefficients using the modified integral indicator I_M. The results of the calculations are presented in the Table. III.

TABLE III. THE VALUES OF THE INTEGRAL INDICATOR AND IDENTIFICATION OF THE ALTERNATIVES' ACCEPTABILITY LEVEL, CALCULATED BY THE METHOD OF DYNAMIC WEIGHTING COEFFICIENTS

Alternatives	Normalized values for partial comprehensive indexes		Values or normalized weighting coefficients		Values of a generalized comprehensive index I_M	The alternative's acceptability level
	F_1	F_2	w_1	w_2		
A_1	0.04	0.21	0.56	0.44	0.159	Very bad
A_2	0.15	0.24	0.51	0.49	0.274	Satisfactorily
A_3	0.27	0.34	0.50	0.50	0.433	Satisfactorily
A_4	0.64	0.00	0.69	0.31	0.622	Satisfactorily
A_5	0.36	0.47	0.50	0.50	0.583	Satisfactorily
A_6	0.50	0.55	0.50	0.50	0.739	Good
A_7	0.42	0.46	0.50	0.50	0.624	Satisfactorily
A_8	0.50	0.47	0.50	0.50	0.681	Good

The analysis of the presented calculations shows that, in this case, there are no alternatives that would have the level of acceptability of "Very good." Therefore, the choice can be made only between two alternatives: A_6 and A_8, which have the acceptability level of "Good," and the values of the generalized efficiency evaluations are close.

The advantage of using the method of dynamic weighting coefficients is that these coefficients are calculated based on the values of partial comprehensive indexes without introducing external information into this procedure. This, in our opinion, increases the objectivity of the obtained results.

V. Conclusion

To describe a system consisting of a set of alternative decisions and the structure of relations between them, it is appropriate to use some mapping that projects it into a numerical system containing a subset of the set of real numbers with an even relation between their elements. This mapping corresponds to the some measurement procedure. The result is some scalar function, which is a comprehensive index. Its formalized description is based on the presentation of the scale as a quantitative carrier of optimal measurement information and the rules of interpretation of this information. The assessment of measurement accuracy by a comprehensive index is characterized by a quality function that reflects the generalized difference in the presentation of the structure of connections between alternatives in the studied and constructed numerical systems.

The model of block convolution of partial criteria in the process of constructing a comprehensive index is considered. A new method of determining the weighting coefficients of partial comprehensive indexes is proposed, named the method of dynamic weighting coefficients. Its feature is the dynamic consideration of a weightier partial comprehensive index obtained at the first stage of convolution, which allows the reduction of the compensatory effect of the second partial comprehensive index at small values of the first one. It is proved that the method satisfies the main properties of the comprehensive index. Practical calculations for the proposed approach were carried out.

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