BASIC CONCEPT OF THE FINITE ELEMENTS METHOD

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Introductions. The finite element method is an effective numerical method for solving engineering and physical problems. Its scope extends from the analysis of stresses in structures to the calculation of complex systems. With its help the motion of the liquid is considered, the gas flow is studied, the problems of electrostatics and lubrication are solved, the oscillations of the systems are analyzed.

Aim. The aim of the work is to study the basic concept of the finite element method with the definition of the main advantages and main disadvantages.

Materials and methods.

The finite element method is a numerical method for solving differential equations found in physics and technology. The emergence of this method is associated with the solution of space research problems in the middle of the 20th century. It was first published by Turner, Cluj, Martin and Topp. This work contributed to the emergence of other works. A number of articles have been published with applications of the finite element method to the problems of structural mechanics and mechanics of continuous media. An important contribution to the theoretical development of the method was made in 1963 by Melosch, who showed that the finite element method can be considered as one of the variants of the well-known Ralley-Ritz method. In structural mechanics, the finite element method minimizes the potential energy by reducing the problem to a system of linear equilibrium equations.

The connection of the finite element method with the minimization procedure has led to its widespread use in solving problems in other fields of technology. The method was applied to problems described by Laplace or Poisson equations. The solution of these equations is also associated with the minimization of some functional. In the first publications with the help of the finite element method the problems of heat distribution were solved. Then the method was applied to the problems of hydromechanics, in particular regarding the movement of fluid in a porous medium.

The scope of the finite element method was significantly expanded when it was shown that the equations that determine the elements in the problems of structural mechanics, heat distribution, hydromechanics, can be easily obtained in particular using the Galorkin method or the least squares method. The establishment of this fact played an important role in the theoretical substantiation of the finite element method, because it allowed to use it in solving any differential equations. It should be noted that more general theoretical justifications exclude the need for variational formulation of physical problems.

As a result, the finite element method from the numerical procedure for solving problems of structural mechanics has become a general method for numerical solution of a differential equation or system of differential equations.

The basic idea of the finite element method is that any continuous quantity, such as temperature, pressure, displacement, etc., can be approximated by a discrete model based on a set of piecewise continuous functions defined on a finite number of subdomains. Piecewise continuous functions are determined using the values of a continuous quantity in a finite number of points in the area under consideration.

In the general case, the continuous value is unknown in advance and it is necessary to determine the value of this value at some internal points of the region. A discrete model, however, is very easy to construct if we first assume that the numerical values of this quantity at each interior point of the region are known. Then you can move on to the general case. Therefore, when building a discrete model of a continuous quantity act as follows - in the study area is recorded a finite number of points called nodes or just nodes; the value of a continuous value at each node is considered a variable to be determined; the area of definition of a continuous quantity is divided into a finite number of sub-areas (elements) that have common nodes and together approximate the shape of the area; only functions in the form of linear, quadratic and cubic polynomials are considered; a continuous quantity is approximated on each element by a polynomial, which is determined by the nodal values of this quantity, for each element is determined by its own polynomial, polynomials are selected so as to maintain continuity along the boundaries of the element.

The division of the area into elements can be done in two ways. You can, for example, limit each element to two adjacent nodes, forming four elements, or divide the area into two elements, each of which contains three nodes.

Each element corresponds to a polynomial determined by the values at the nodal points of the element. In the case of dividing the area into four elements and each element has two nodes, the function of the element will be linear by x because two points uniquely define a straight line. The final approximation will consist of four piecewise linear functions, each of which is defined on a separate element.

Another way of dividing a region into two elements with three nodal points leads to the representation of the function of the element in the form of a polynomial of the second degree. In this case, the final approximation will be a set of two piecewise continuous quadratic functions. Note that this approximation will be piecewise continuous, because the angles of the graphs of both of these functions may have different values in the third node.

In the general case, the temperature distribution is unknown and it is necessary to determine the value of this value at some points. The method of constructing a discrete model remains as described above, but with the addition of one additional step. The many nodes and temperature values in these nodes are determined again, which are now variable because they are unknown in advance.

The area is divided into elements, each of which determines the corresponding function of the element. Nodal values must now be "adjusted" so as to ensure the "best" approximation of the actual temperature distribution. This "regulation" is carried out by minimizing some value associated with the physical nature of the task. If the problem of heat propagation is considered, then the functional associated with the corresponding differential equation is minimized. The process of minimization is reduced to the solution of systems of linear algebraic equations with respect to nodal values.

When constructing a discrete model of a continuous quantity defined in a two- or three-dimensional domain, the basic concept of the finite element method is used similarly. In the two-dimensional case, the elements are described by functions of coordinates, with the most common elements in the form of a triangle or quadrilateral. The functions of the elements are now depicted as flat or curved surface. The function of an element will be represented by a plane if the minimum number of nodal points is taken for this element, which is equal to three for a triangular element and four for a quadrangular one.

If the number of nodes used is greater than the minimum, the function of the element will correspond to the curved surface. In addition, the excess number of nodes allows you to consider elements with curvilinear boundaries. The final approximation of a two-dimensional continuous quantity will be a set of piecewise continuous surfaces, each of which is determined on a separate element using the values at the respective nodes.

An important aspect of the finite element method is the ability to select a typical element from a set of elements when determining the function of the element. This allows you to determine the function of the element regardless of the relative position of the element in the general connected model and other functions of the elements. Setting the function of an element through an arbitrary set of nodal values and coordinates allows you to use the functions of the element to approximate the geometry of the area.

The scope of the finite element method is very large and covers all physical problems that can be described by differential equations. The most important advantages of the finite element method, due to which it is widely used, are the following - the properties of materials of adjacent elements do not have to be the same, which allows it to be applied to bodies with a complex internal material structure; the curvilinear region can be approximated by means of rectilinear elements or described precisely by means of curvilinear elements; the sizes of elements can be variable; it is possible to apply boundary conditions with discontinuous surface load, as well as mixed boundary conditions.

Results and discussion. The main concepts of the finite element method with the definition of the main advantages and main disadvantages are considered in the work.

Conclusions. The finite element method is an effective numerical method for solving engineering and physical problems, and the scope covers a wide range of problems.