MACHINE IMPLEMENTATION OF THE FINITE ELEMENT METHOD

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Introductions. The procedure for compiling a system of equations, its subsequent transformation and further solution with a general calculation scheme is considered.

Aim. The purpose of the work is to consider the issues of constructing a global stiffness matrix and a system of linear equations, a general calculation scheme and a machine implementation of the problem solution.

Materials and methods. The use of the finite element method leads to a system of algebraic equations, the order of which coincides with the total number of unknowns. The number of unknowns, and hence the order of the system, is usually very large, and a computer is needed to solve such systems.

Some methods for constructing a global stiffness matrix are very inefficient for machine implementation. This inefficiency is explained by the fact that the stiffness matrix of an individual element has the same number of rows and columns as the global stiffness matrix. Most of the coefficients in the element matrix are zero. Additional inconveniences are associated with the fact that the global stiffness matrix is obtained by summing the stiffness matrices of the elements. The matrix of each element must be calculated separately from and then added to the last one, and this requires memorizing both matrices. The need to remember two large matrices leads to memory overload when the problem being solved has a large number of unknowns.

In efficient programs, the procedure for constructing a global stiffness matrix uses a reduced form of element matrices when deriving equations for an element. This method is known as the "direct stiffness" method. The use of this method eliminates the need to store large matrices of elements containing only a few non zero coefficients.

The direct stiffness method for constructing the global stiffness matrix is a very important machine implementation algorithm for the finite element method, because it significantly reduces the memory load. In particular, it eliminates the need to remember large element matrices that contain only a few non-zero coefficients. The number of rows and the number of columns of the reduced stiffness matrix of the element are equal to the number of degrees of freedom of the element.

When using the finite element method, a system of linear equations is obtained, which must be solved with respect to unknown nodal parameters. Solving these equations is a very important aspect of the problem as a whole, because the system of equations is usually very large. Methods for solving systems with a small or large number of equations differ little from each other. Machine implementation of these methods, however, depends on the technical possibilities.

When considering the discretization process of a continuous medium, it was noted that by proper numbering of nodes, one can control the location of the coefficients in the global stiffness matrix. Recall that with a reasonable numbering of nodes, a band-type matrix is obtained instead of a full matrix. A band matrix is characterized by the fact that all its non-zero coefficients are located near the main diagonal, and all coefficients outside a certain strip bounded by lines parallel to the main diagonal are equal to zero.

Two properties of the resulting system of equations make it ideal: symmetry and positive definiteness of the matrix. The presence of symmetry means that approximately half of the non-zero terms of the matrix can be omitted. Positive definiteness means that the coefficient on the main diagonal is always positive and is usually much larger in magnitude than any other coefficient in the corresponding row or column.

In the case of a symmetric positive-definite band-type matrix, the amount of calculations required to obtain a solution to the system of equations is significantly reduced. It also reduces the likelihood of large rounding errors.

The existence of symmetry in a band-type matrix makes it possible to significantly reduce the amount of memory required to store the global matrix. Usually, when programming, the transformation of the matrix into a rectangular array is provided, the width of which coincides with the width of the matrix strip, and the length is equal to the number of equations.

One of the most effective methods for solving a system of equations that are obtained using the finite element method is the well-known version of the Gauss elimination method. The system matrix is converted to a triangular form, after which the solution is obtained by inverse sweep. The method includes two stages. The first is to transform the original matrix into a triangular one. At the second stage, the resulting system of equations is solved. The first step is usually called matrix decomposition, since the stiffness matrix is converted into a simpler matrix. The second stage of the solution is called backward sweep.

One of the advantages of the finite element method is that many of its steps are common to all areas of application of the method. The procedure for solving problems of heat transfer and groundwater flow includes many of the same steps that are encountered in the calculation of rigid frames and trusses and in the analysis of the stress and strain states of a deformable continuous medium.

All programs that implement the finite element method must contain preliminary information about the number of equations, the number of elements, and the bandwidth of the matrix. Information about the number of equations is necessary so that in the initial state the global stiffness matrix and the global load vector can be filled with zeros (preliminary cleaning of the matrices), since these matrices are summed up during the calculation.

Immediately after cleaning the matrices, a cyclic operation is performed for each element. This operation includes entering initial information about the element, compiling element matrices and including them in global matrices. Specific element information includes the element number and node numbers. This may include the values of the coordinates of the nodes of the element. The latter can be entered independently and called from the machine memory using the node numbers of the element. If the latter procedure is used, all nodal coordinates must be entered before starting the loop.

After the transformation of the system of equations, these equations are solved with respect to unknown nodal values. There are several procedures for constructing a solution. One of them has already been discussed in the middle of this chapter. The system of equations has a special form: its matrix is banded, and the diagonal elements are usually positive and dominate over the elements of the corresponding columns and rows outside the main diagonal. This allows many fairly general solution procedures to be modified in such a way as to increase their efficiency.

After solving the system of equations, the output of the nodal values is carried out. If element results are not calculated, then this stage is final.

One more element loop is required to calculate element results. In this loop, the element's input data is entered again, the element's results and all other important quantities associated with the element are calculated. If the element information is not stored in an external peripheral, then using read statements identical to those used in the initial data entry has some merit. This allows the same set of inputs to be used to calculate element results. You can also include comparison operators in your program that will compare the computed values with the maximum or minimum values for previous elements and leave the smallest or largest value along with the element number.

Printing information of this type is made after considering all the elements.

An efficient program does not consider the global stiffness matrix, the global load vector, and the decision vector as separate arrays with predetermined sizes, but stores all these values in a common one-dimensional array as a column.

Preparing the initial data requires a lot of attention from the programmer. Most often, the incorrect operation of the program is due to errors in the source data. Before sending the initial data to the computer, it is necessary in some way to make sure that they are correct. There are several ways to do this: some of them are very simple, others are more complex.

Results and discussion. The procedure for compiling a system of equations, its subsequent transformation and further solution with a general calculation scheme is considered.

Conclusions. The paper considers the issues of constructing a global stiffness matrix and a system of linear equations, a general calculation scheme, and a machine implementation of solving the problem

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