LINEAR INTERPOLATION POLYNOMS, FINITE ELEMENTS METHOD AND BORDER PROBLEMS

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Introductions. In the finite element method, the procedure of minimization of the integral quantity, which is caused by the corresponding physical process, can be used to determine the nodal values. This value can be the potential energy of the system, as in the problems of structural mechanics. Minimization of the potential energy of the system allowed reducing the calculations to the solution of the system of algebraic equilibrium equations with respect to possible nodal displacements. In some cases, the system of computational equations is obtained by minimizing the functional for which the minimization function satisfies the original differential equations and boundary conditions.

Aim. The aim of the work is to consider the application of the finite element method to the solution of boundary value problems when the calculated equations are obtained by minimizing some integral quantity.

Materials and methods. Consider a one-dimensional heat flow in a rod with a heat-insulated side surface, when a heat flux of a certain intensity brings to its fixed end, and at its free end there is convective heat transfer with a given heat transfer coefficient and at a known ambient temperature. Since the rod has a heat-insulated side surface, heat exchange does not occur through it.

The differential equation for the temperature distribution inside the rod with given boundary conditions has a single solution. This equation, in the end, allows us to obtain a numerical solution.

The problem of heat transfer can also be solved using a variational approach. In variational calculations to minimize the functional, it is necessary to satisfy the differential equation with respect to the boundary conditions. This, in turn, allows you to determine the temperature at each node. To minimize the functionality, various functions of the elements are used, which are defined on a single element and expressed through nodal values. These nodal values are unknown quantities. They determine the value of the functional. Minimization of this functionality is performed on these values.

The initial stage of implementation of the finite element method is to determine the subdomains and their nodal points. The rod can be divided into linear elements with nodal temperature values. The temperature inside the elements is determined by the appropriate ratios. For the heat transfer problem of the example, the functional is the sum of volume and surface integrals. Surface integrals are fairly easy to compute because subintegral expressions correspond to nodal values. The volume integral contains the temperature derivative and must be divided into two integrals, since this derivative does not maintain continuity over the volume of the body as a whole. When calculating the integral, it is assumed that the cross-sectional area of each element is constant. Representation of the volume integral by region as the sum of integrals, by individual elements, allows you to define different material properties for different elements. This is an important feature of the finite element method.

As a result, we obtain the expression for the functional through the nodal values of temperature. The correct temperature values correspond to the minimum value of the functional.

The last stage of the analysis is the assignment of appropriate values for the physical characteristics of the material and obtaining numerical values of temperature.

Results and discussion. In the finite element method, the procedure of minimization of the integral quantity, which is caused by the corresponding physical process, can be used to determine the nodal values. This value can be the potential energy of the system, as in the problems of structural mechanics. Minimization of the potential energy of the system allowed to reduce the calculations to the solution of the system of algebraic equilibrium equations with respect to possible nodal displacements. In some cases, the system of computational equations is obtained by minimizing the functional for which the minimization function satisfies the original differential equations and boundary conditions.

Conclusions. The paper considers the application of the finite element method to the solution of boundary value problems when the calculated equations are obtained by minimizing some integral quantity.

Literature

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