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MECHANICS OF MACHINES

Increase in Solution Stability of Ill-Conditioned Dynamics Problems

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Abstract—The difficulties of design determination of the quantitative characteristics of weight, elasticity, damping, critical frequencies of fast-revolving rotors are demonstrated. It is suggested to use two approaches in order to obtain accurate and stable solutions of ill-conditioned reverse problems of rotor dynamics. The first one concerns the reduction of design model size. It is shown that relatively uncomplicated design rotor models are used as they were considered the most effective and stable, and match the selected design scheme and parameter values (hardness, weights, deflections, etc.) obtained as a result of experiments or calculations. The second approach deals with statistical method of the increase of solution stability. Its efficiency was shown while identifying rotor eccentricity by the example of five-mass rotor model.

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INTRODUCTION

Compressor rotor of gas-turbine engine AI-20 of a disk-drum type consists of ten separate disks carrying rotor blades on its rim, rear rotor shaft, and unit sealing labyrinths of rear and front bearing (Figure).

Turbine torque is received by rear shaft splines and is transmitted from disk to disk by radial pins located in slots under blades. Compressor disks are joined by over-pressing along coupling belts with a given tightness. All disks and compressor rear shafts after mechanical processing undergo static balancing in dynamic conditions, and assembled compressor–rotor dynamic balancing up to a final imbalance of 5 g cm per each support.

Exploiting engines AI-20 and carrying out long tests on test benches revealed the following defects: bending of rear shaft, damage of font junction of compressor rotor, mainly, joining rear shaft with the tenth rotor and also a number of defects of combustion-chamber casing.

It was assumed that the given defects are connected with the passage through critical frequencies of compressor rotor spinning in the system of the engine; the assumption gained strength due to the extremely high level of rotations of engines AI-20 on idling when fluctuations are resonant.

The engines were designed so that their resonant modes and critical frequencies of rotations were beyond the areas of exploitation frequencies. That means that the problems of design determination of bend shape and critical rotary frequencies of aircraft engine mounted on plane wing are very difficult and often fail. This is explained by the necessity of the involvement of joint-bearing masses, bodies, engine bearer, and wing because of the radiation of fluctuations of different parts, having an amortizing effect on the one hand and the other hand exciting fluctuations.

It is very complicated task to determine the quantitative specifications of weight, elasticity, damping, etc. using the design of a modern engine not having an existing prototype.

RESEARCH TASK STATEMENT

The calculation of the first critical frequency of compressor rotations conducted in accordance with the Stodola method, which considered shear forces but disregarded the compliance of stage junctions with each other due to the absence of reliable data, showed that it is equal to 22150 r/min. The critical rotation frequency, determined with the integral method without consideration of shearing forces, was 25400 r/min. The specified critical rotation frequency was determined regarding local compliance and was equal to 19400 r/min. The calculation of local compliance was conducted with the use of the experimentally found compliance of a compressor rotor in an average cross-section. Finally, considered support

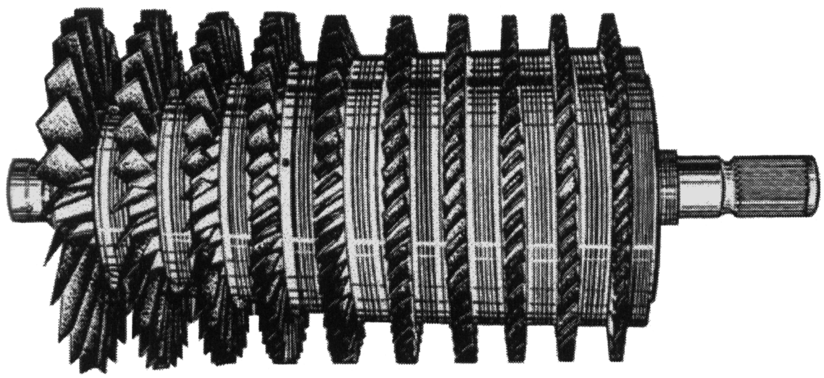

Figure.

compliance critical rotation frequency was 15800 r/min, and 16600 r/min, with regard to hydroscopic analysis. Experiments showed that the value of real critical velocity was 11200 r/min.

Such a considerable difference in calculations and real values caused the necessity to develop the methods of statement and solution of inverse problems concerning the identification of elastic centrifugal and dissipative machine characteristics in accordance with the results of corresponding experiments when required parameters are counted regarding all important machine peculiarities. The value of the identification parameters involves determination with the use of accepted idealization of real object, i.e., brought to a selected mathematical model describing a real system.

ELIMINATION OF POOR CONDITIONALITY OWING TO REDUCTION OF DESIGN MODEL SIZE

In order to specify the values of critical rotation frequency and to determine the hardness characteristics of compressor rotors of engine AI-20, we conducted a static test of several similar rotors.

To determine static coefficients of rotor influence (compliance) static tests of it were carried out. Coefficients of influence α_{ij} were equal to value of deflection in section i from action of individual force in section j: $i, j = 1, 2, ..., 10$, were determined according to the technique described in [1].

The critical frequency of serial compressor rotor of engine AI-20 on rigid support was estimated with the use of static coefficients of influence and was within the limits from $n_{cr} = 11200$ to $n_{cr} = 12200$ r/min. Thus, it may change by 9–10% depending on the tensions of coupling belts of disks and pins and also on the rigidity of sections between rotor stages.

The low value of the critical rotation frequency $n_{cr} = 11200$ r/min was obtained for rotor no. 603255 with minimal tensions on coupling belts of disks and pins, and the upper one, for spent rotor no. 8808.

At the same time, the matrix of coefficients of influence $\mathbf{A}$ with a size of 10×10 and condition number cond($\mathbf{A}$) = 3696 was ill-conditioned. Consequently the possible maximal error of determination of critical frequency may be 1000%.

Naturally, larger size n of matrix $\mathbf{A}$ the greater the model corresponds with reality, but in such case cond($\mathbf{A}$) grows and solution error increases. Thus, on the one hand, approximating a real rotor with a large number of concentrated weights we bring a design scheme to real construction and, correspondingly, increase the accuracy of rigidity occurrence or influence coefficients and the whole design, but, on the other hand, design errors increase. This contradiction is a result of the identification of the imperfection of the algorithm, and it should be considered while substantiating the acceptability of the described identification of flexible rotor imbalances and on the basis of balancing method errors, choosing a reasonable number of disks approximating a real rotor. This emphasizes the fact that relatively simple design models are considered the most acceptable and stable variants. They differ as they require parameter values that are accurate and equivalent to the given scheme obtained as a result of experiment: rigidity, weights, and deflection. Significant results confirming this assumption are obtained in the work [2], offering different reduction methods of design model dimension based on the usage of analytical criteria of weak interactions within the system. It allows the discovery of a small number of variables (or subsystems) determining system dynamics in the particular range of frequencies. The resulting minimal design models have parameters equal to the initial system and obtained on the basis of initial elastic centrifugal parameters. [3] shows that reduced design models are described by well-substantiated matrices.

In the present article, a rotor was schematized with one point weight equal to rotor weight applied in its center of mass with rigidity in the affixment point equal to rotor rigidity in the section containing the center of mass. It is shown that such simple schematization does not affect the value of critical rotation frequency considerably in comparison with one calculated for multi-disk rotor if real values of rigidity and weights are used in both schemes.

To calculate ω_{cr} it is necessary to solve the following equation:

$$1 - m_1\alpha_{11}\omega^2 = 0, \quad \text{or} \quad n_{cr} = 300/\sqrt{\delta_{cm}}\ \text{r/min}, \tag{1}$$

where δ_{cm} is the deflection of rotor weight.

The center of gravity of compressor rotor of engine AI-20 is located between the fifth and sixth stages, $l_{cg} = 43.74$ cm. According to the static rotor loads, the influence coefficient is selected for crossing of mass center, $\alpha_{cg} = 6.35 \times 10^{-6}$ cm, $\delta_{cm} = 6.35 \times 115.4 \times 10^{-6} = 770 \times 10^{-6}$ cm, where $G = 1154$ N is the weight of compressor rotor. Consequently, $n_{cr} = 11\,200$ r/min.

The more accurate value calculated for the rotor considering the influence coefficient of all stages with the use of the Rayleigh method is 11 220 r/min. Thus, the schematization error of a tenth-mass rotor with one-mass unit was not more than 0.4% and in calculations to determine ω_{cr1} rotor can be considered as one-mass system using accurate values of rigidity and weight.

STATISTICAL METHODS OF REDUCTION OF POOR CONDITIONALITY

The stated inverse identification problem can be solved accurately without simplification of the mathematic model to one mass using special methods to increase the accuracy of solutions of ill-conditioned equation systems. We also developed a statistical method based on the fact that input parameters, determined experimentally, are regarded as random values and distributed according to particular laws [4].

The mathematical expectation of these values is accepted as their real values in assumption that change error contains only random component and does not contain systematic component and mean square deviation is value $\Delta/3$, where Δ is the accuracy of measuring device. Then unknown characteristics are also random values distributed according to particular laws.

With growth in n, the value of mathematical expectation will approach to its real value, but relative errors of measured values will decrease due to the accuracy of determination of required values. The most effective method is carried out along with one or some known methods of conditional number reduction, as necessary measurement of values decrease as well.

The algorithm of search of unknown values can be different. For example, the right parts of system equations are measured some times and every time unknown values are found. Then the mathematical expectation of the latter ones is determined after processing obtained data with the use of statistical methods, or the right parts are measured in different conditions. As before, unknown numbers are found, or various object parameters expressed on the basis of the same characteristics are measured and then these unknown characteristics are determined. Their mathematical expectations are accepted by different random values and calculated.

All these algorithms differ in efficiency and we should give preference to the algorithm that can be carried out more easily and involves the law of value distribution. The values are considered to be random, which allows a reliable evaluation of their distribution reliably.

We will then analyze the usage of this idea to determine unknown rotor eccentricities with influence coefficients by solving the matrix equation

$$\mathbf{y} = \mathbf{A}\omega^2(\mathbf{y} + \mathbf{e}), \tag{2}$$

where $\mathbf{y}$ is the vector of rotor deflection in n of its sections measured on the rotation frequency ω, $\mathbf{e}$ is the vector of rotor eccentricities in the same sections, and $\mathbf{A}$ is the matrix of influence coefficients α_{ik} and weight m_k located in the products of studied sections.

The work [5] shows the dependences of relative errors of eccentricities on the relative errors of deflection measurement, and the influences on the coefficients of matrix and rotation frequency expressed on the basis of number of matrix substantiation.

According to the dependencies we studied a five-mass model whose masses are given by matrix of products $\alpha_{ik} \times m_k$, $i = 1, 2, ..., 5$, $k = 1, 2, ..., 5$.

$$\begin{bmatrix} 8.3 \times 10^{-8} & 7.4 \times 10^{-8} & 7.7 \times 10^{-8} & 6.0 \times 10^{-8} & 4.9 \times 10^{-8} \\ 6.7 \times 10^{-8} & 9.1 \times 10^{-8} & 11.0 \times 10^{-8} & 9.1 \times 10^{-8} & 9.2 \times 10^{-8} \\ 4.8 \times 10^{-8} & 7.8 \times 10^{-8} & 12.0 \times 10^{-8} & 10.7 \times 10^{-8} & 12.0 \times 10^{-8} \\ 3.8 \times 10^{-8} & 7.0 \times 10^{-8} & 10.6 \times 10^{-8} & 12.2 \times 10^{-8} & 15.8 \times 10^{-8} \\ 2.4 \times 10^{-8} & 4.7 \times 10^{-8} & 9.4 \times 10^{-8} & 11.8 \times 10^{-8} & 19.0 \times 10^{-8} \end{bmatrix}, \ \mathrm{s}^{-1}.$$

Critical rotation frequencies on hard supports are equal to 14000, 28900, 65300, 130600, 419300 r/min. The number of matrix substantiation cond(**A**) ≈ 900.

To check the efficiency of the method a numeral experiment was conducted. On the basis of accurate values of section eccentricities (77.4×10^{-4}, 89.9×10^{-4}, 105.0×10^{-4}, 79.0×10^{-4}, 59.5×10^{-4} cm) accurate values of rotor deflection were determined by solving equation (2), where matrix **A** did not have errors. These values $\tilde{\mathbf{y}}$ were regarded as mathematical expectations of deflections in the given sections. Further, stating the mean square deviation MSD $\sigma = \Delta/3$, where Δ is the accuracy of measurements using computer generator of random numbers, different deflections were obtained as random values distributed according to regular distribution law with particular parameters. This experiment stipulated the generation of 50 deflections in each section. For each deflection we found realizations **e** and on this basis, their mathematical expectations. At the same time, an experiment was conducted using different measurement accuracies, and, correspondingly, different mean square deviations.

The calculation showed that when measurement accuracy was 0.1 dispersion of required values reached 500%, in case when accuracy was 0.01–200%, and 0.001–100%. Otherwise, as a result of dispersion of measured values, the error of determination of required eccentricities may reach 400–500% depending on measurement accuracy. At the same time, eccentricities were determined with the use of the suggested method and differed from real values by 15% when measurement accuracy was 0.1, by 2% at 0.01 and by 0.5% at 0.001.

These figures show the high accuracy of the described solution of identification problem. The obtained results prove the efficiency of the statistic algorithm application to identify objects and processes accurately.

CONCLUSIONS

It was established that simple design models of fast rotors are considered to be the most available and stable. They differ as they use accurate and equivalent parameters obtained as a result of experiments: rigidities, weights, deflections.

It was shown that ten-mass rotor can be presented as one mass model and give accurate results for the accurate solution of a problem if it contains accurate data of model parameters equivalent to the given calculation scheme.

Also the statistic method was offered; it increases the stability of solutions of ill-conditioned equation systems and shows their efficiency during the identification of rotor eccentricities.

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