Active Noise and
Vibration Control Methods
Krakow - Krasiczyn, Poland
June, 11 - 14, 2007
www.vibrationcontrol.pl

EVOLUTION OF STICK-SLIP REGIME AND CHAOS FOR FRETTING-PROCESS

Jurij SHALAPKO [1], Sergej KOSTOGRYZ [2]
[1,2] Department od Mechanics
Khmelnitskij National University,
Khmelnitckiy,Ukraine,290016
shal@beta.tup.km.ua, shalapko@yahoo.com.ua

Keywords: dynamic system, friction, fretting, stick-slip

1. Introduction

Damage, such as loss of material or/and cracking, induced by the small amplitude oscillatory relative movement between contact surfaces of contacting components is usually termed fretting [1]. The modeling of dynamic systems with friction for fretting processes is one of the most important and difficult problems of mechanical science and engineering. Two main categories of problems are noticed: the first one concerns "microscopic" friction models and is representative for the tribology and general contact theory. The second one concerns "macroscopic" descriptions of friction actions (stick-slip phenomena) in discrete systems with friction. The term slip-stick refers to the motion of objects where self-excited vibrations occur due to successive slipping and sticking at a frictional interface. Slip-stick is present in a huge variety of mechanical systems from micro to geological length scales. The phenomenon has been studied from a variety of perspectives over the past 75 years [2].

Theoretical and experimental works (Den Hartog, 1930; Hundal, 1979; Shaw, 1986) on research frictional oscillators have shown that in simple mechanical systems there are complex dynamic phenomena's, which description becomes by complicated display of instability and occurrence of chaotic vibrations.[3,4,5]. Complex evolution of dynamic systems with friction is a result nonlinearity of friction and also is the direct by a source of transfer of motion from one body to another through contact interaction into interface. This is interaction it is simultaneously accompanied of wear processes of surface layer , changes of tangential stiffness, normal pressure, amplitude of relative displacement, and together with it lead to change for relative velocity of slipping. Accepting in attention the corrosion in contact that are described by the phenomenon of fretting, it is possible to assert that about change of friction force due to corrosion phenomena as a result of morphological changes for surface layer. Dynamic of friction processes at excitation of a contact zone result to noise the phenomena which are self-excited vibrations. The phenomenon of squeal is also associated with stick- slip. This is a term first used by Bowden and Tabor [6] when describing the relative motion of two surfaces in contact. They noted that the motion is governed by a kinematic friction law while the surfaces are slipping and by a static law when there is no relative motion - sticking.

Andronov A.A, considers, that the main reason of instability of system is the falling characteristic of friction [7]. Kononenko V.O. [8] has presented kinetic model of friction and its feature. Modern models and mechanisms of transition from static friction to dynamic are designed on use of computer techniques in iterative processes [9]. In many models the key moments which describe friction in conditions of instability is [10, 11, 12, 13. 14]:

- dependence of friction on relative velocity,
- effect of memory at friction or time lag of critical slip amplitude behind value of friction force,
- uncertainty or a polisemanticy of friction forces at zero relative velocity,
- time delay at sticking,
- preliminary displacement or pre-sliding.

In the present research the two-of-freedom model of dynamic interaction is submitted at kinematic excitation of the interface in view of adjusting parameters: amplitudes and mass loss of one of a surface. The choice of of adjusting parameters is determined destruction by fretting – processes. At cyclic loading of contact surfaces the slip amplitude increases on according to of a degree of destruction of contact spots. Thus, the history evolution of dynamic system with dry friction and fretting can assume the following stages: full sticking of surfaces, partial slip in borders of a spot of contact, a mode of stick-slip and gross sliding of surfaces.

2. Evolutionary model of frictional interaction for small amplitude fretting

Let's define integrity of nominal - foxed joints of two surfaces as relative mobility of its surface layers. The model will consist of two body (m <<M), one of which describes direct contact, and another bodyt of a detail as a whole (Fig. 1).

Well-known, that properties of a surface layers considerably differ from properties of a material into volume. According to Rabinowihcz [15], the static friction reaches the greatest value at the maximal elastic - plastic deformation is thin - a surface layer, commensurable with the sizes of asperity, and then smoothly falls

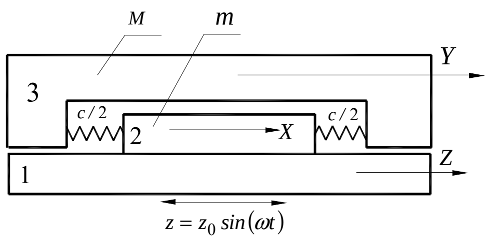

Fig. 1 Differentiated model of frictional interaction of two surfaces with two active mass.

under the certain law with increase in relative slip velocity. It is necessary to notice, that growth of force friction from zero till the moment if slip occurs quickly enough in comparison with all period of relative displacement of surfaces, that is typical for small amplitude fretting (10…30 μm). On Fig. 2 the behaviour of the suggested model is illustrated at sign-variable motion of a foundation with separate motion of small under-contact zones and all body as a whole. For high - frequencies oscillations of a foundation about position of the maximum static force of friction the stick condition is unstable enough as force of static friction is not determined at zero relative velocity.

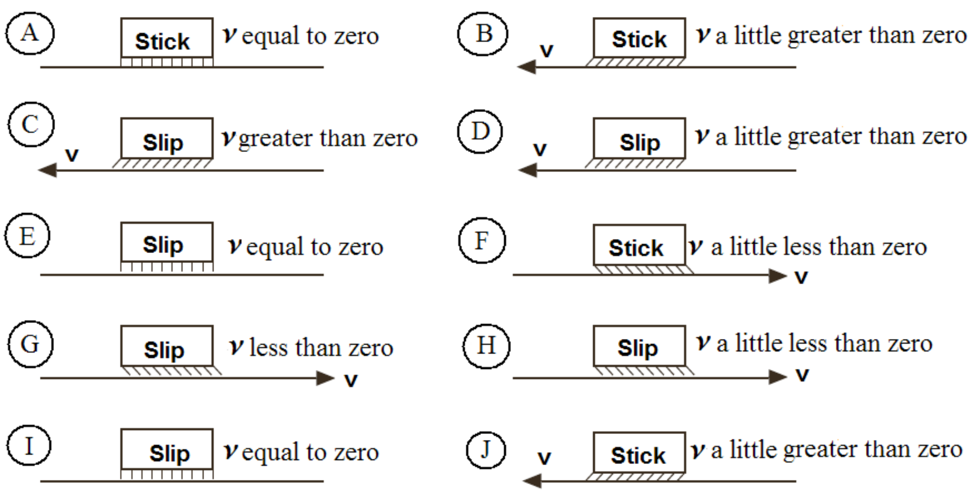

Fig.2. The scheme of deformation of a surface layer at small tangential motion.

Considering such physical and mechanical factors as increase of quantity of joints for micro asperity, loss of stiffness of a surface layer because of a cyclic tangential loading, wear, fluctuations of slip amplitude and time of sticking can to assume, that in system with two degrees- of- freedom and above named control parameters it is necessary to expect chaotic friction in the interface.

Uncertainty of conditions can track on Fig. 2. So, for same conditions B and D, F and H two conditions are possible: stick or slip. At kinematics excitation bases Z (Fig. 1) friction force acts as driving force of process. Therefore, definition of the law of friction depending on relative micro-displacement and relative velocity is the main task in the decision of system of the differential equations:

$$\begin{cases} mx'' = -c(x-y) + F_j \\ My'' = c(x-y) \end{cases} \qquad (1)$$

where c - tangential stiffness of a surface layers;

F_j - friction force which is a constant on very short a j - time interval between $\lfloor t_{j-1}, t_j \rfloor$. From the second equation of system (1) have:

$$x = y + \frac{M}{c} y''; \tag{2}$$

$$x'' = y'' + \frac{M}{c} y^{(4)}. \tag{3}$$

Let's substitute the equation (2), (3) in the first equation of system (1):

$$m\left(y'' + \frac{M}{c} y^{(4)} \right) = -c\left(y + \frac{M}{c} y'' - y \right) + F_j; \Rightarrow \frac{mM}{c} y^{(4)}$$
$$+ (M+m) y'' = F_j; \Rightarrow y^{(4)} + \frac{M+m}{mM} c y'' = \frac{c}{mM} F_j \tag{4}$$

Let's enter a designation:

$$a^2 = \frac{M+m}{mM} c = (M+m) b \ , \qquad b = \frac{c}{mM} \tag{5}$$

and (4) we shall write down as:

$$y^{(4)} + a^2 y'' = bF_j \tag{6}$$

The decision (6) search as:

$$y = y_o + y_r \ ,$$

where y_0 - the decision of the homogeneous differential equation

$$y^{(4)} + a^2 y'' = 0 \ ,$$

y_r- the private decision of the equation (6).

The decision of the characteristic equation $k^4 + a^2 k^2 = 0$ there will be a following:

$$k_1 = k_2 = 0 \, , k_{3,4} = \pm a_i \left(i^2 = -1 \right)$$

The decision of the homogeneous equation we shall write down as

$$y_o = C_1 + C_2 t + C_3 \cos at + C_4 \sin at \tag{7}$$

The private decision:

$$\begin{gathered} y_r = At^2 \quad (A = const) \\ y'_r = 2At; \quad y''_r = 2A; \quad y'''_r = y_r^{(4)} = 0 \end{gathered} \tag{8}$$

From (6) it is received

$$a^2 \cdot 2A = bF_i \quad , \quad A = \frac{bF_i}{2a^2}$$

The common decision of the differential equation (6):

$$y = C_1 + C_2 t + C_3 \cos at + C_4 \sin at + At^2 \tag{9}$$

$$y'' = -C_3 a^2 \cos at - C_4 a^2 \sin at + 2A \tag{10}$$

From (3):

$$\begin{aligned} & x = C_1 + C_2 t + C_3 \cos at + C_4 \sin at + \\ & + At^2 + \frac{M}{c}(-C_3 \cos at - C_4 \sin at)a^2 + 2A\frac{M}{c}; \Rightarrow \\ & x = C_1 + 2A\frac{M}{c} + C_2 t + At^2 + \left(1 - a^2\frac{M}{c}\right)(C_3 \cos at + C_4 \sin at). \end{aligned} \tag{11}$$

Definition of constants C_1, C_2, C_3, C_4 everyone j- time interval is a separate mathematical problem, and was considered in [16]. Thus, the friction force has been determined at the moment of stick surfaces in depending on inertion of small surfaces layers(m) , detail (M) and deformation of a spring (c):

$$F_j = \frac{a^3 c}{b \sin(a\Delta t)\left(\Delta tac + Ma^2 - c\right) + \Delta tac} \cdot \\ \cdot \left[\begin{aligned} & V\omega \cos\left(\omega t_j\right) + \left(\frac{c}{a^2 M} - 1\right) y'_{j-1} - \frac{c}{a^2 M} x'_{j-1} + \left(\frac{c}{aM} - a\right)\left(y_{j-1} - x_{j-1}\right) \cdot \\ & \cdot \sin(a\Delta t) + \left(1 - \frac{c}{a^2 M}\right)\left(y'_{j-1} - x'_{j-1}\right)\cos(a\Delta t) \end{aligned} \right], \tag{12}$$

where $V\omega \cos\left(\omega t_j\right) = z'_j$ - velocity of a foundation during the moment t_j

The behaviour of dynamic system is investigated for each cycle $t_j = j \cdot \Delta t \quad (j = 1,2,.....N)$ separately during time $t = \Delta t \cdot N$, with the purpose of definition of the slip beginning . Then friction force is determined by function of relative velocity (fig. 3):

$$T(vslip) := \begin{cases} -To \cdot \left[1 - \left(\dfrac{vg - |vslip|}{vg}\right)^n\right] & \text{if } vslip < 0 \\ To \cdot \left[1 - \left(\dfrac{vg - |vslip|}{vg}\right)^n\right] & \text{if } (vslip \geq 0) \\ \left(a \cdot vslip^2 + b \cdot |vslip| + To + 2\right) \cdot sign(vslip) & \text{if } |vslip| \geq vg \end{cases} \quad , (13)$$

where vg - relative velocity of surfaces in conditions of pre-sliding displacement,
vslip – relative velocity of slipping,
n - parameter of plasticity at pre-sliding displacement,
a, b - factors of Stribek-effect.

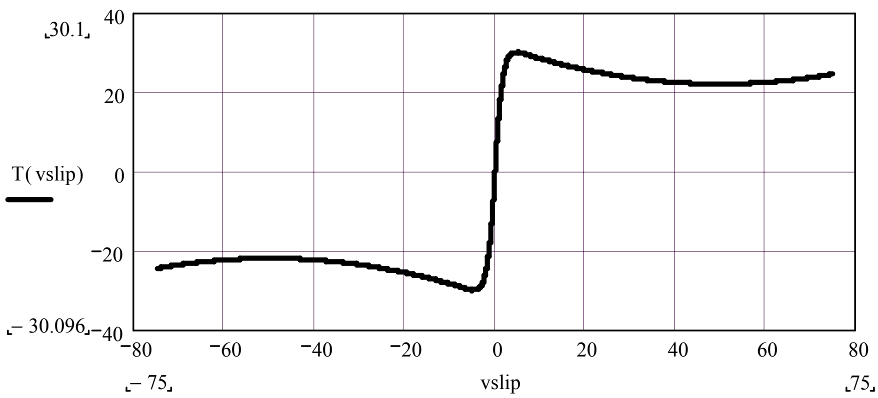

Fig. 3. The characteristic of friction

3. Results of modelling.

Let's consider process of transition from full stick two bodies prior to the beginning slip for parameters for exciting frequency of oscillation 100 Hz, mass ratio $\boldsymbol{m/M} = 2 \cdot 10^{-6}$, static friction $\boldsymbol{F_{max}}$ =10H , amplitude of oscillation of a foundation 10 µm. On fig. 4 the diagram of relative motion of a body on a foundation is shown. It is visible, that the slip regime occurs to the beginning of motion at achievement to the maximum amplitude of displacement. The phase portrait shows, that the seating of a stick - slip regime has been progressive character and within an 5-7 periods proceed (fig. 5). On fig. 6 value of slip velocity $\dot{x} - \dot{z}$ is shown. Having define value of velocity and frequency of act microslip it is possible to define size of dissipation of energy in the interface depending on control parameters of process. So in steady regime which is shown on fig. 4 velocity slip achieves 2,6 mm/s. At known relative velocity, microdisplacement, times of stick and slip it is possible to define a deformation of friction component for surface layer and frictional for the interface.

$$F(t) = F_{def} + F_{fr} = c[x(t) - y(t)]_{stick} + F\ [z(t) - x(t)]_{slip}$$

The first summand defines contact fatigue of surface layer, cracking and stratification, the second – process of wear.

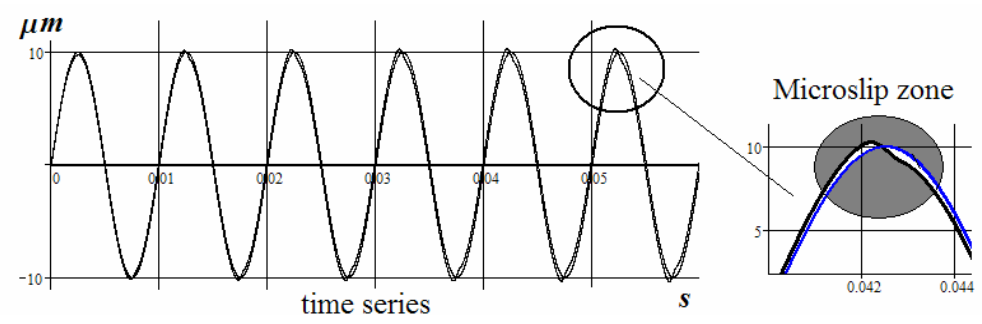

Fig. 4. The oscillogram of displacements of a foundation 1 and a body 2.Amplitude of oscillation of a foundation of 100 Hz, amplitude 10 μm, friction force 10N, m/M=2·10^{-6}.

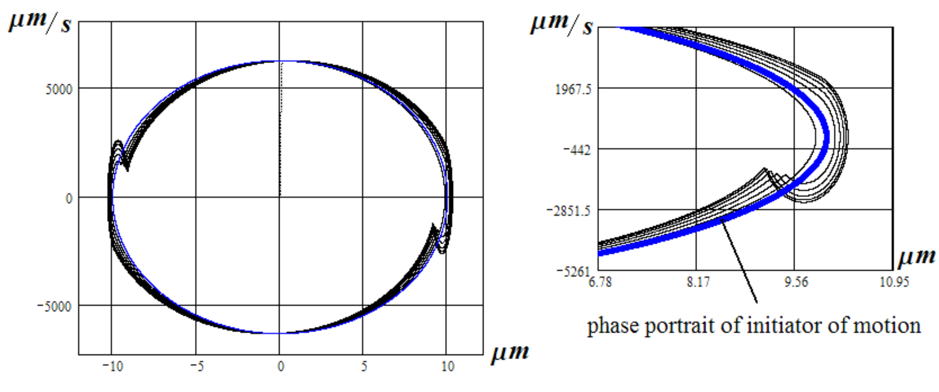

Fig. 5. A phase portrait of oscillatory process at initial stick-slip process

On fig. 7,8 the oscillogram, a phase portrait and is relative velocities for decreased of mass body 3 by 10 % (m/M=1,8·10^{-6}). The maximum of slip velocity achieves already

11 mm/s, which considerably increase a frictional component of friction. It is necessary to notice, that self-oscillatory process in the interface is initiated also.

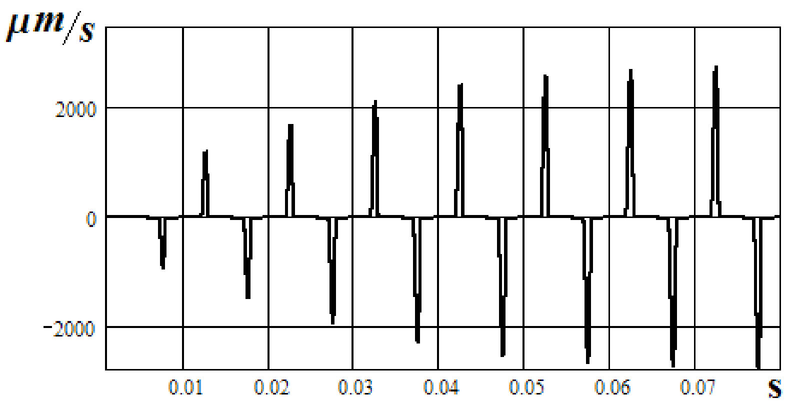

Fig.6. Relative velocities of slip for m/M=2·10^{-6}.

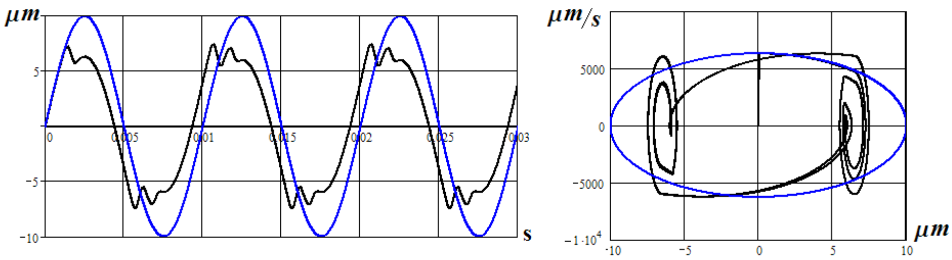

Fig.7. Oscillatory processes at decreased of mass 2 by 10 % *(*m/M=1,8·10^{-6}*).*

Because of the big mass a body 3 is practically motionless. However, at increase of mass M or increased exciting frequency the system behaves is unpredictable. The history of transition from quasi - periodic motion occurs on the mechanism of doubling of the period. The decreasing mass 1, that is the simulation of wear and deformation of surface layer, it is possible to define whether the system of stationary, periodic or chaotic behaviour in some region of space of parameters does not discover and also to define. And also to make sure that the system does not contain the hidden external or internal sources of accidental veritable noise. For the design of behaviour of surface of section of interface as a dynamic system the following parameters at zero initial conditions were accepted: tangential stiffness of mobile surface layer 12.8 N/μm; parameters of description of friction $a = 0.00024$, $b = -6$; friction force $F = 64N$, amplitude of excitation 12 μm; frequency vibrations 100 Hz. Mass of the deformed layer 0.003 kg, mass of greater body

3.5 kg. Such distributing of the masses describes the very real picture: so at the contact area 0,01 m^2 the thickness of elastic-deformed layer will be 35…38 μm. We explore evolution through phase portraits, the Poincare maps and the Fourier transform of time- series.

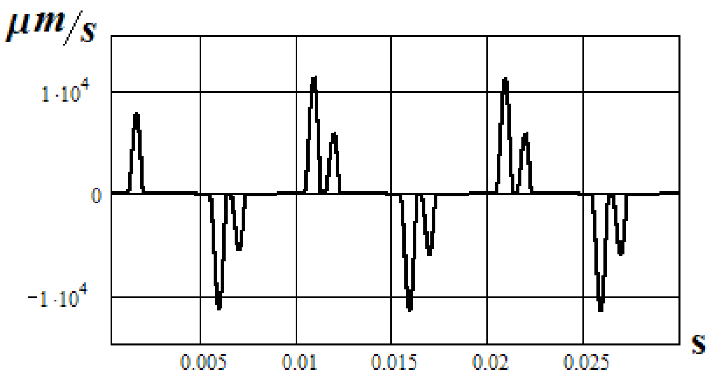

Fig. 8. Relative of slip velocities at decreased of mass 2 by 10 % *(*m/M=1,8·10^{-6}*)*.

The decreasing mass 1, that is the simulation of wear and deformation of surface layer, it is possible to define whether the system of stationary, periodic or chaotic behaviour in some region of space of parameters does not discover and also to define. And also to make sure that the system does not contain the hidden external or internal sources of accidental veritable noise. For the design of behaviour of surface of section of interface as a dynamic system the following parameters at zero initial conditions were accepted: tangential stiffness of mobile surface layer 12.8 N/μm; parameters of description of friction $a = 0.00024$, $b = -6$; friction force $F = 64N$, amplitude of excitation 12 μm; frequency vibrations 100 Hz. Mass of the deformed layer 0.003 kg, mass of greater body 3.5 kg. Such distributing of the masses describes the very real picture: so at the contact area 0,01 m^2 thickness of elastic-deformed layer will be 35…38 μm. We explore evolution through phase portraits, the Poincare maps and the Fourier transform of time- series(fig.9,10,11). On fig. 9,10 trajectories of motion of a body 2 and phase portraits for different mass 2are shown.

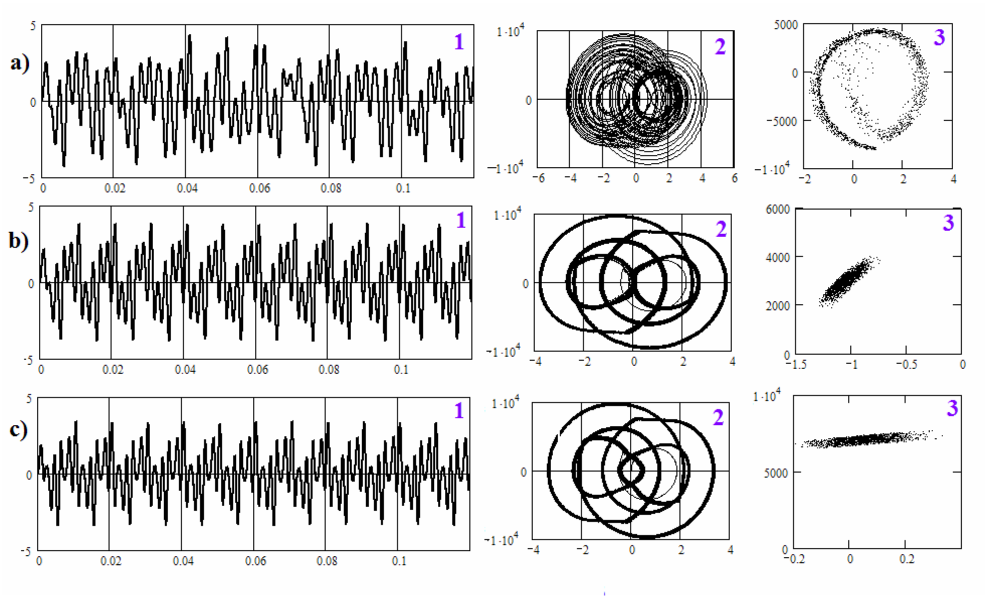

Fig.9. Time-series, phase portraits and Poincare maps for m=0.0046 kg (a), m= 0.004 kg (b), m= 0.0035 kg(c)

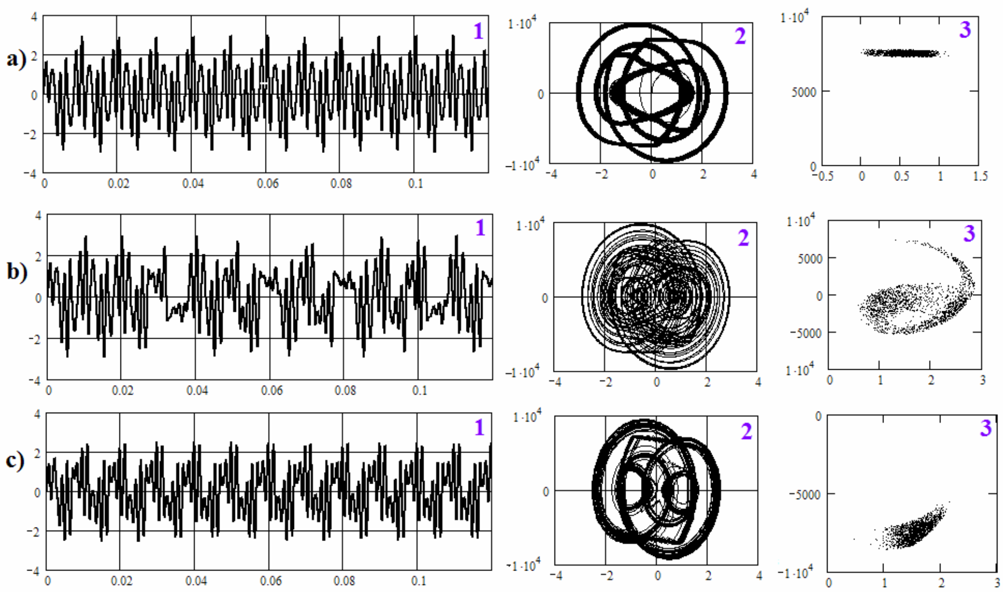

Fig.10. Time-series, phase portraits and Poincare maps for m=0.00305 kg (a), m=0.003 kg (b),m= 0.0027 kg(c)

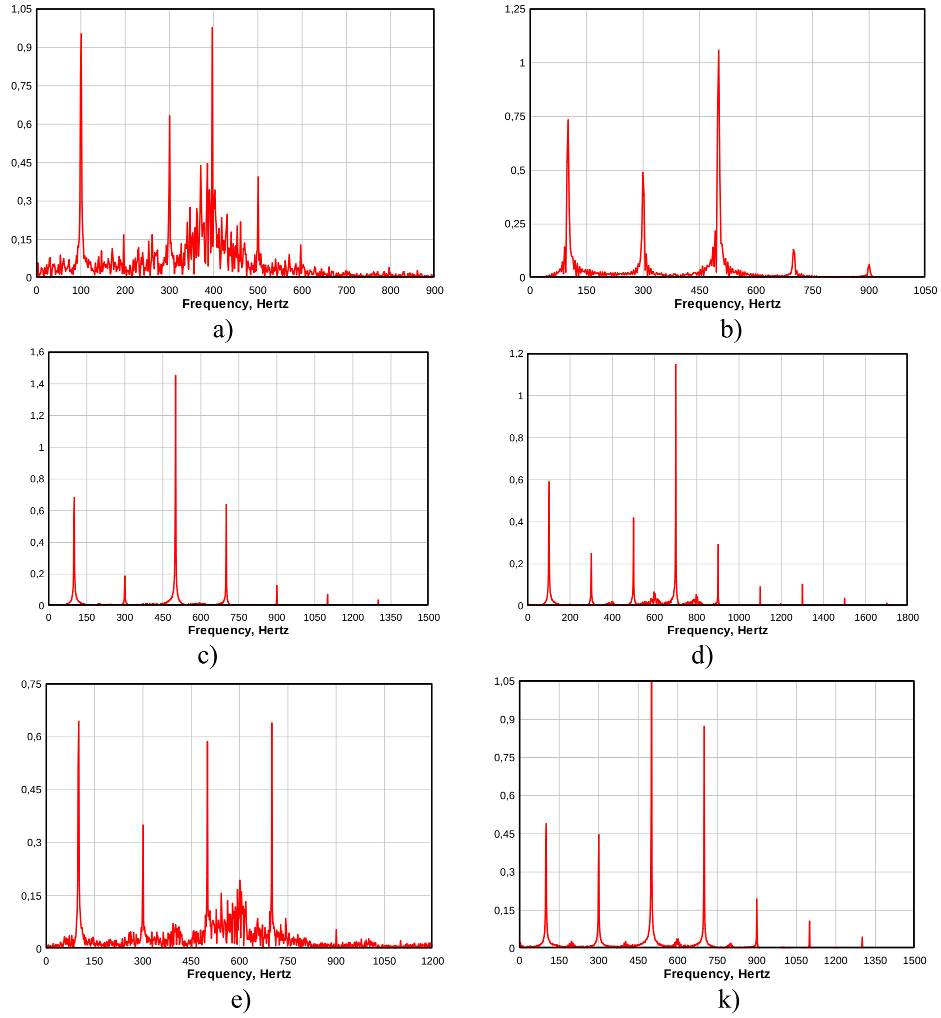

Fig.11. Fourier transform of time- series for different mass 2: m=0.0046 kg(a),m=0.004 kg(b),m=0.0035 kg(c), m=0.00305 kg(d),0.003 kg(e),0.0027 kg(k)

Conclusions

Dynamic model of nominal-fixed frictional joints to become unsteady at the change the masses of the deformed surface layer. System at first to be found in the stationary state (steady stick-slip), but after the change of the mass 2 is the row of the unstable states. By the precursor of chaotic motions of presence two and more periodic vibrations. The Poincare maps for such quasi-periodic motion is the reserved curve on a phase portrait (fig.9). With loosening of motions enter into action of nonlinear - motion goes out on a limit cycle. Such transitions is name a Hopf bifurcation . As a numeral experiment shows, the process of loss of mass for a surface layer especially in the case of existence of the brightly expressed of hardness layer causes jumping change of character of relative motion of surfaces, for example at *m*=0,003 g and *m*=0,00305g. Such change of dynamic behaviour of the system at the small change of control parameter of *m*, testifies to exponential divergence of two near position of trajectories. The Fourier spectrums testify also to the three frequency mechanism of origin of chaos at fretting is possible.

References

[1] Waterhouse RB. Fretting fatigue. London: Applied Science, 1981.

[2] Ibrahim R. Friction-induced vibration, chatter, squeal, and chaos. Part II. Dynamics and modeling. Appl. Mech. Rev. 1994; 47:227–53.

[3] Den Hartog, J.P., Forced vibration with combined Coulomb and viscous damping, Trans. of. Amer. Society of Mech. Eng. – 53-1930-pp.107-115

[4] Hundal, M.S., 1979, Response of a base excited system with Coulomb and viscous friction, Journal of Sound and Vibration – 64 – pp.371-378.

[5] F. P. Bowden and D. Tabor 0853 The friction and lubrication of solids, Oxford University Press.See Part II, Chapter IV.

[6] Shaw, S.W., 1986, On the dynamic response of a system with dry friction, J. of Sound and Vibration – 108(2) – pp. 305-325/

[7] Andronov, A.A., Vitt, A.A. and Khaikin, S.E., *Theory of Oscillations*, Pergamon Press, 1966

[8] Kononenko, V.O., Frictional auto-oscillations close to harmonic. Transactions of civil engineering institute of USSR Academy of Science 1954, N9

[9] Shalapko Y. Design and features of friction in transition from stick to slip// Transactions Technological University ,Chmelnickij, 2007,1,234-241

[10] R. S. H. Richardson and H. Nolle. Surface friction under time-dependent loads// Wear-1976-37:87–101

[11] A J McMillan , A Non-Linear Friction Model For Self-Excited Vibrations// Journal of Sound and Vibration –1997- 205(3)-323-335

[12] D. D. Rizos and S. D. Fassois, Presliding friction identification based upon the Maxwell slip model structure// Chaos- 2004-v. 14- no. 2- pp. 431–445.

[13] Karnopp, Dean Computer Simulation of Stick-Slip Friction in Mechanical Dynamic Systems// Transactions of the ASME-1985- Vol 107

[14] Leine, R.I., Campen, D.H., de Kraker, A.,Van den Steen, L., Stick-Slip Vibrations Induced by Alternate Friction Models// Nonlinear Dynamics -1998-16, 41-54

[15] E.Rabinowicz The nature of the static and cinematic coefficients of friction // Journal of Applied Physics, 22(11), 1951

[16] Shalapko Y.I. Evolution model of frictional interaction of surface layer for fretting. Transactions of engine of construction -2006.-№ 4 - c.44- 49