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A PRESTRESSED ELASTIC STRIP WITH ELASTIC REINFORCEMENTS

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A contact problem is studied for a prestressed elastic strip with an elastic reinforcement. The integral Fourier transform is used to construct an influence function for an infinite strip with one face fixed. A unit concentrated force is applied to the strip at an arbitrary angle. The contact problem on force transfer from a thin infinite stringer to the prestressed strip is solved. The problem is mathematically formulated as a system of integro-differential equations for the unknown contact stresses on the assumption that the beam bending model and the uniaxial stress model are valid for the stringer, which is subjected to both vertical and horizontal forces. This system is solved in a closed form using the integral Fourier transform. The contact stresses are expressed in terms of Fourier integrals in a quite simple form. The influence of the initial stresses on the contact stress distribution is analyzed, and effects of concentrated load are revealed.

Introduction. Elastic contact problems on load transfer from elastic plates, stringers, or coatings of various shapes and dimensions to massive bodies without initial stresses were touched on in a great many studies whose results are presented in the monographs [1, 11]. Contact problems for elastic compressible and incompressible bodies with homogeneous initial stresses are studied in [2, 12–14] using specific elastic potentials.

The present paper provides a linearized elastic formulation for some contact problems on a prestressed elastic strip interacting with elastic reinforcements. The study is made in a common form for compressible and incompressible bodies, based on the results from [3, 10].

1. Problem Formulation and Initial Resolving Equations. It is assumed that an elastic strip with initial stresses is reinforced with elastic pieces of very small thickness h, which are subjected to vertical and horizontal forces. Based on the results from [11], we assume that the standard beam bending model and the uniaxial stress state model are valid for the elastic reinforcements, i.e., they are bent as ordinary beams in the vertical direction and are compressed or stretched as rods in a uniaxial stress state in the horizontal direction. We also assume that an elastic reinforcement (an infinite stringer, a finite plate, etc.) has finite flexural stiffness, i.e., their elastic moduli E_1 are greater than the elastic moduli E_2 of the prestressed strip, which, incidentally, follows from the smallness of the thickness h.

Based on the aforesaid, let us analyze the equilibrium of an elastic reinforcement on the section $-\infty < y_1 < \infty$.

Assuming that the cross section of the reinforcing element is a rectangular with a unit width and using Hooke's law, we find the axial stress acting along the Oy_1-axis:

$$\sigma_{y_1 y_1}(y_1) = E_1 \varepsilon_{y_1 y_1}, \tag{1.1}$$

$$\varepsilon_{y_1 y_1} = \frac{du(y_1)}{dy_1}. \tag{1.2}$$

Here $u(y_1)$ is the horizontal displacement of the elastic reinforcement.

The equilibrium conditions for the reinforcing element are

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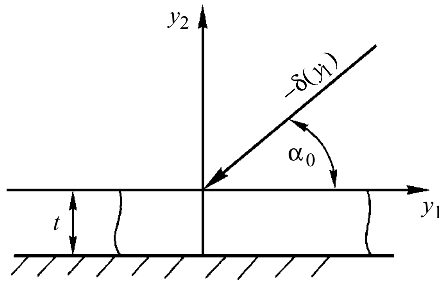

Fig. 1

$$\sigma_{y_1 y_1}(y_1) = \frac{1}{h} \int_{-\infty}^{y_1} [q(t) - q_0(t)] dt. \tag{1.3}$$

From (1.1)–(1.3), we find

$$\frac{du(y_1)}{dy_1} = \frac{1}{E_1 h} \int_{-\infty}^{x} [q(t) - q_0(t)] dt \quad (-\infty \le y_1 \le \infty). \tag{1.4}$$

From the assumption that the reinforcing element is bent as an ordinary beam in the vertical direction, we may write

$$D \frac{d^4 u(y_1)}{dy_1^4} = P(y_1) - P_0(y_1) \quad (-\infty \le y_1 \le \infty). \tag{1.5}$$

Here $u(y_1)$ is the vertical displacement of the reinforcing element, D is the flexural stiffness of the rod, and $P_0(y_1)$ and $P(y_1)$ are the intensities of the vertical forces.

The displacements remain continuous at the interface between the elastic reinforcement and the prestressed elastic strip, i.e.,

$$u(y_1) = u_1(y_1), \quad u(y_1) = u_2(y_1), \quad -\infty < y_1 < \infty, \tag{1.6}$$

where $u_1(y_1)$ and $u_2(y_1)$ are the displacements in the strip.

It is necessary to evaluate the effect of the initial stresses on the distribution of the normal, $p(y_1)$, and tangential, $q(y_1)$, contact stresses along the reinforcement–strip contact line. To this end, we will determine the influence function that is due to the concentrated unit normal or tangential forces applied to the face of the strip (Fig. 1), i.e., we will determine the fields of elastic displacements and stresses in the strip.

2. Influence Function for the Prestressed Strip. Let us adhere in our study to the general linearized theory of elasticity with arbitrary elastic potentials used in the theory of large (finite) initial strains. In passing to various versions of the theory of small initial strains, the simplifications mentioned in [2] should be introduced.

Following [3–9], we use the solutions for compressible and incompressible bodies in the coordinates of the initial strain state y_i, which are related to the Lagrange coordinates (of the natural state) by $y_i = \lambda_i x_i$ $(i = 1, 2)$, where λ_i are the elongations determining displacements of the initial state. All quantities associated with the elastic reinforcements will be written in the notation accepted in the theory of elasticity, and all quantities associated with the strip will be denoted as in [3–9]. Let the initial state of the strip be homogeneous and the following conditions be satisfied [2]:

$$u_m^0 = (\lambda_m - 1) x_m, \quad \lambda_m = \text{const}. \tag{2.1}$$

Here u_m^0 is the displacement determining the initial state when the initial stresses are homogeneous.

Let us write boundary conditions for a unit force $\delta(y_1)$ applied at an angle α_0 to the upper face of the prestressed strip. From Fig. 1, we find