LINEAR INTERPOLATION POLYNOMS

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Introductions. The basis of the finite element method is the idea of approximating a continuous function of a physical quantity by its discrete model. Such a model is built on a set of piecewise continuous functions. These functions are defined on a finite number of subdomains. Such subdomains are called elements. A polynomial is most often used as such an approximating element function. The order of the polynomial is determined by the number of continuous function data at each element node.

The classification of finite elements can be carried out in accordance with the order of the polynomial functions of these elements. In this case, the following three groups of elements are considered: simplex, complex and multiplex elements.

Simplex elements correspond to polynomials containing a constant and linear terms. The number of coefficients in such a polynomial is one more than the dimension of the coordinate space. The polynomial of the simplex function for a two-dimensional triangular element is linear and contains three coefficients, according to the number of nodes.

Polynomial functions are used to describe complex-elements. They contain a constant, linear terms, and, if necessary, terms of the second, third, and higher order. Complex-elements have additional boundary nodes and may have internal nodes. The number of nodes in the complex-element is greater than the dimension of the coordinate space plus one. The interpolation polynomial for a two-dimensional triangular complex-element includes six coefficients, so the element under consideration must have six nodes. The shape of the complex elements can be the same as that of the simplex elements.

For multiplex elements, polynomials with high-order terms are used, but the boundaries of the elements must be parallel to the coordinate axes in order to achieve continuity when moving from one element to another. The boundaries of simplex and complex elements are not subject to this restriction.

Aim. The aim of the work is to consider the use of linear interpolation polynomials as the basis of the finite element method.

Materials and methods. A one-dimensional simplex element is a straight line segment of a certain length with two nodes, one at each end of the segment. The origin of the coordinate system is located outside the element. A polynomial function for a scalar value has the form of a linear polynomial with one constant and one linear term. The polynomial coefficients can be determined using the conditions at the nodal points.

A two-dimensional simplex element is a triangle with straight sides and three nodes, one at each vertex. The logical numbering of element nodes is required, for example, counterclockwise, starting from some arbitrary node. The interpolation polynomial for a two-dimensional simplex element has the form of a linear polynomial with one constant and two linear terms. The polynomial coefficients can be determined using conditions at the nodes.

Triangular finite elements have a number of features. The function of the field of the studied physical quantity changes linearly between any two nodes. And since nodes define the boundaries of an element, which are straight lines, this function changes linearly along each of its three sides. Thus, it is fair to note that any line along which the field function of the physical quantity under study takes the same values is a straight line that intersects the two sides of the element, except for the case when the values of this function are the same at all nodes. These features of triangular finite elements make it easy to define scalar level lines.

A three-dimensional simplex element is a tetrahedron. Its four nodes are indicated by indices, and the nodes are traversed counterclockwise. The interpolation polynomial for a two-dimensional simplex element has the form of a linear polynomial with one constant and three linear terms.

Previously considered interpolation relations for a scalar quantity. For vector quantities, more than one unknown must be determined at each node. In this case, the vector quantity is represented by its components, which are considered as unknown scalar quantities. Depending on the dimension of the problem, each node will contain one, two or three unknowns.

To obtain a system of equations for unknown values of nodal quantities, it is necessary to perform integration over the area of the element of the shape functions or their partial derivatives. If the interpolation relations are written in the coordinate system associated with the element, then the integration procedure is simplified. The coordinate system associated with an element is called the local coordinate system. Interpolation relations in the local coordinate system can be written by transforming the corresponding equations obtained in the global coordinate system.

In the case of a one-dimensional element, it is possible not to use the local coordinate system, since the interpolation equation is quite easily integrated. Some simplification of the integration operation is achieved by placing the origin of the local coordinate system at the 1st node of the element.

For a triangular element, the most common is the natural coordinate system, which is defined by three relative coordinates. Each such coordinate is the ratio of the distance from the chosen point of the triangle to one of its sides to the height lowered to this side from the opposite vertex. The value of the relative coordinate varies from zero to one. Relative coordinates are called$L$-coordinates. The values of these coordinates give the relative values of the areas of the triangles into which the element under consideration is divided. The advantage of using$L$-coordinates is the presence of integral formulas that simplify the calculation of integrals along the sides of the finite element and, also, over its area.

The natural coordinate system for a tetrahedral element is introduced similarly to the case of planar$L$-coordinates. The four relative distances are defined as the ratio of the distances from a selected arbitrary point of an element to one of its faces to the height lowered to this face from the opposite vertex. Such$L$-coordinates are called volumetric. For a tetrahedral element, the sum of all four relative coordinates is equal to one.

Polynomial equations that are used to approximate scalar and vector quantities within an element therefore give correct results when the nodal values of the considered quantities are equal to each other and ensure continuity in the interelement zones.

The solution obtained by the finite element method will converge to the exact solution with decreasing element dimensions, provided that, as soon as the nodal values are equal to each other, the interpolation equations lead to constant values of the considered quantities within the element, while the gradients are assumed to be infinitely small.

The interpolation equations for an element must model constant values if such values occur. Such criteria impose certain restrictions on the shape functions, namely that the sum of the values of the shape functions must be equal to one at each interior point of the element. If the specified criterion is not met, then the polynomial approximation will not give constant values, even if, by condition, they should be.

A discrete model for a continuous function is built on a set of piecewise continuous functions, each of which is defined on a separate element. Further, for the integration of a piecewise continuous function, the condition of its continuity in the interelement zone is formulated. The integral of a step function is defined insofar as the function remains bounded. For the integral to be defined, the function must be continuous together with its derivatives up to a certain order, inclusive, which ensures that the derivative of the corresponding order has only a finite number of step-type discontinuities. Compliance with this condition means that the first partial derivatives of the approximation function must be continuous on the boundaries between the elements if the differential equation contains second-order partial derivatives. All used differential equations are presented in the form of relations that contain no more than first partial derivatives, therefore, interpolation functions should be required to be continuous in the interelement zone, but their partial derivatives are not required to obey this condition. Continuity for a one-dimensional element is guaranteed, since any two adjacent elements have a common node.

Results and discussion. The basis of the finite element method is the idea of approximating a continuous function of a physical quantity by its discrete model. Such a model is built on a set of piecewise continuous functions. These functions are defined on a finite number of subdomains. Such subdomains are called elements. A polynomial is most often used as such an approximating element function. The order of the polynomial is determined by the number of continuous function data at each element node.

The classification of finite elements can be carried out in accordance with the order of the polynomial functions of these elements. In this case, the following three groups of elements are considered: simplex, complex and multiplex elements.

To obtain a system of equations for unknown values of nodal quantities, it is necessary to perform integration over the area of the element of the shape functions or their partial derivatives. If the interpolation relations are written in the coordinate system associated with the element, then the integration procedure is simplified. The coordinate system associated with an element is called the local coordinate system. Interpolation relations in the local coordinate system can be written by transforming the corresponding equations obtained in the global coordinate system.

Conclusions. The robot considers one-, two-, and three-dimensional simplex elements, describes the procedure for interpolation of vector quantities, presents the concept and describes the local coordinate system and the properties of the interpolation polynomial.

Literature

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