

## CONTACT PROBLEM FOR HALF-SPACES AND ELASTIC CYLINDER WITH INITIAL (RESIDUAL) STRESSES\*

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The contact problem of the interaction of two prestressed half-spaces with a prestressed elastic cylindrical punch without friction forces is solved in the case of unequal roots of the characteristic equation. The study is conducted in a common form for the theory of large initial (final) strains and two variants of the theory of small initial strains within the framework of the linearized theory of elasticity for an arbitrary elastic potential. The problem is solved using the Hankel transform, paired integral equations, orthogonal polynomials, and other methods of the theory of contact problems of linearized elasticity. The numerical analysis is conducted for the Treloar potential in the form of curves.

**Keywords:** linearized theory of elasticity, initial (residual) stresses, contact problem, cylindrical punch, half-space

**Introduction.** An urgent task of modern mechanical engineering and construction is to increase the durability, wear resistance, and reliability of mechanisms, machines, and engineering structures. The wide range of scientific research in the field of solid mechanics contributes to the successful solution of this problem. A number of factors such as increasing the service life of machines and structures by improving their operating characteristics, reducing their weight, reducing the cost of materials, and their economic compatibility depend on the methods of calculating load transfer in machine parts and structures.

Many publications in periodicals and educational and monographic works [1, 17, 19] address issues related to contact problems for elastic, plastic, and viscoelastic bodies without initial (residual, process-induced) stresses. The number of such publications is quite large. However, modern engineering practice put forward a number of tasks that require the use of complicated nonclassical models. Such models include those that take into account the complex physical and mechanical properties of bodies [8, 10, 11–15]. The factors that should be taken into account during the contact interaction of solids include properties related to the micromechanics of frictional interaction, such as the surface properties of the material, friction, surface hardness, heat generation, wear resistance of the surface of bodies, etc.

In addition to the above factors, it is important to take into account the initial stresses. Despite the fact that the development of contact problems has made significant progress in recent decades, there are still a number of problems in solid mechanics that require the attention of the scientific community.

Since initial stresses are present in virtually all components of machine and structure parts, buildings, and structure elements, we list a number of actions that contribute to their occurrence: material production processes, technological

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## REFERENCES

1. D. V. Grilitskii and Ya. M. Kizima, *Axisymmetric Contact Problems of the Theory of Elasticity and Thermoelasticity* [in Russian], Vyshcha Shkola, Lviv, (1981).
2. A. Guz, S. Babich, and Yu. Glukhov, *Mixed Problems for a Prestressed Elastic Foundation* [in Russian], Lambert Academic Publishing (2015).
3. A. N. Guz and V. B. Rudnitskii, *Basic Theory of Contact of Elastic Bodies with Initial (Residual) Stresses* [in Russian], Mel'nik, Khmelnytskyi (2006).
4. L. M. Filippova, "Spatial contact problem for a prestressed elastic body," *Prikl. Mat. Mech.*, **42**, No. 6, 1080–1084 (1978).
5. V. M. Aleksandrov and N. K. Arutyunyan, "Contact problems for prestressed deformed bodies," *Sov. Appl. Mech.*, **20**, No. 3, 209–215 (1984).
6. S. Yu. Babych and Y. P. Glukhov, "On one dynamic problem for a multilayer half-space with initial stresses," *Int. Appl. Mech.*, **57**, No. 1, 43–52 (2021).
7. S. Yu. Babych and N. O. Yarets'ka, "Contact problem for an elastic ring punch and a half-space with initial (residual) stresses," *Int. Appl. Mech.*, **57**, No. 3, 297–305 (2021).
8. O. M. Bagno, "Effect of finite initial strains on the wave process in the system of an incompressible half-space and an ideal liquid layer," *Int. Appl. Mech.*, **57**, No. 6, 644–654 (2021).
9. R. S. Dhaliwal, B. M. Singh, and J. G. Rokne, "Axisymmetric contact and crack problems for a initially stressed Neo-Hookean elastic layer," *Int. J. Eng. Sci.*, **18**, No. 1, 169–179 (1980).
10. A. N. Guz, *Eight Non-Classical Problems of Fracture Mechanics*, Springer, Cham (2021).
11. A. N. Guz and A. M. Bagno, "Influence of prestresses on normal waves in an elastic compressible half-space interacting with a layer of a compressible ideal fluid," *Int. Appl. Mech.*, **55**, No. 6, 585–595 (2019).
12. A. N. Guz and A. M. Bagno, "Influence of prestresses on quasi-lamb modes in hydroelastic waveguides," *Int. Appl. Mech.*, **56**, No. 1, 1–12 (2020).
13. A. N. Guz, "Nonclassical problems of fracture/failure mechanics: on the occasion of the 50th anniversary of the research (review) III," *Int. Appl. Mech.*, **55**, No. 4, 343–415 (2019).
14. A. N. Guz, "On general list of references to the monograph "Eight non-classical problems of fracture mechanics,"" *Int. Appl. Mech.*, **58**, No. 1, 1–29 (2022).
15. A. O. Kaminsky and E. E. Kurchakov, "Tensor-nonlinear constitutive equations for an elastic body with primary anisotropy," *Int. Appl. Mech.*, **58**, No. 2, 154–159 (2022).
16. M. Kurashige, "Circular crack problem for initially stressed neo-Hookean solid," *ZAMM*, **49**, No. 8, 671–678 (1969).
17. G. B. Muravskii, "Response of an elastic half-space with power-law nonhomogeneity to static loads," *Arch. Appl. Mech.*, No. 78, 965–987 (2008).
18. V. B. Rudnitskii and N. N. Dikhtyaruk, "Interaction between an infinite stringer and two identical prestressed strips: contact problem," *Int. Appl. Mech.*, **53**, No. 2, 149–155 (2017).
19. N. P. Semenyuk and N. B. Zhukova, "Stability of a sandwich cylindrical shell with core subject to external pressure and pressure in the inner cylinder," *Int. Appl. Mech.*, **56**, No. 1, 40–53 (2020).
20. N. A. Yaretskaya, "Contact problem for the rigid ring stamp and the half-space with initial (residual) stresses," *Int. Appl. Mech.*, **54**, No. 5, 539–543 (2018).