UNIFIED ALGORITHMS OF KINETOSTATIC ANALYSIS OF SECOND-CLASS LINKAGE MECHANISMS USING MATHCAD

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Linkage mechanisms have wide application in different fields of machinery due to their advantages in comparison with the other types of mechanisms: they are composed by the rigid bodies and only lower kinematic pairs with the geometric closure of the links, which provide more durability and reliability of the mechanisms. Besides, it enables to transmit higher loads and higher working velocities and thus provides more productivity of machines.

Kinetostatic analysis is an important task to design a linkage mechanism properly. It is known that this task can be solved using traditional methods of the theory of mechanisms and machines. Besides, modern CAD systems can be also used. So, the simplest way to make kinematic and kinetostatic analysis of a linkage mechanism is to make its 3D-model and use mechanical CAD/CAE system (for example, SOLIDWORKS Motion, etc). But to perform a complex optimization procedure, such way is not always the best solution.

In this paper we consider another method to solve this task by means of Mathcad using unified algorithms. It is known that linkage analysis can be done by making unified programs for certain kinematic chains – structural groups [3-5, 7]. For kinetostatic analysis, we used an analytical approach described in [7].

In this paper we consider Mathcad algorithms for analysis of the structural Assur groups of II class, 1st and 2nd types. Notice that unified algorithms for the other types of kinematic chains of II class are also developed by the author and used in the academic process in the course of “Theory of Mechanisms and Machines” at the faculty of Mechanical Engineering, as was described in [4].

Assur group of the 1st type. The scheme for kinetostatic analysis of this group is shown on Fig. 1, a. The links of that group are loaded with the forces $\bar{F}_i, \bar{F}_j$, the moments M_i, M_j, the gravity forces $\bar{G}_i, \bar{G}_j$. The inertia forces

$\bar{F}_{\text{ін}i}, \bar{F}_{\text{ін}j}$ and the moments of inertia forces $M_{\text{ін}i}, M_{\text{ін}j}$ of the links i and j are calculated using known kinematic characteristics of the structural group:

$$F_{\text{ін}ix} = -m_i x''_{Si}\omega_1^2; F_{\text{ін}iy} = -m_i y''_{Si}\omega_1^2; F_{\text{ін}jx} = -m_j x''_{Sj}\omega_1^2;$$
$$F_{\text{ін}jy} = -m_j y''_{Sj}\omega_1^2; M_{\text{ін}i} = -J_{Si}\varphi''_i\omega_1^2; M_{\text{ін}j} = -J_{Sj}\varphi''_j\omega_1^2. \quad (1)$$

All the forces are divided into components which are directed along the coordinate axes. Firstly, define the reaction force $\bar{R}_{ji}$ in internal kinematic pair:

$$R_{ji} = \sqrt{R_{jix}^2 + R_{jiy}^2};\ R_{jix} = \left(A_1 l_j \cos\varphi_j + A_2 l_i \cos\varphi_i\right) / l_i l_j \sin\left(\varphi_i - \varphi_j\right);$$
$$R_{jiy} = \left(A_1 l_j \sin\varphi_j + A_2 l_i \sin\varphi_i\right) / l_i l_j \sin\left(\varphi_i - \varphi_j\right). \quad (2)$$

The coefficients A_1 and A_2, in (2) are defined as:

$$A_1 = M_i + M_{\text{ін}i} - F_{ix} MK_i \sin\left(\varphi_i + \gamma_{Ki}\right) + F_{iy} MK_i \cos\left(\varphi_i + \gamma_{Ki}\right) + \left(F_{\text{ін}iy} - G_i\right)\times$$
$$\times MS_i \cos\varphi_i - F_{\text{ін}ix} MS_i \sin\varphi_i; A_2 = M_j + M_{\text{ін}j} - F_{jX} NK_j \sin\left(\varphi_j + \gamma_{Kj}\right) +$$
$$+ F_{jy} NK_j \cos\left(\varphi_j + \gamma_{Kj}\right) + \left(F_{\text{ін}jy} - G_j\right) NS_j \cos\varphi_j - F_{\text{ін}jx} NS_j \sin\varphi_j. \quad (3)$$

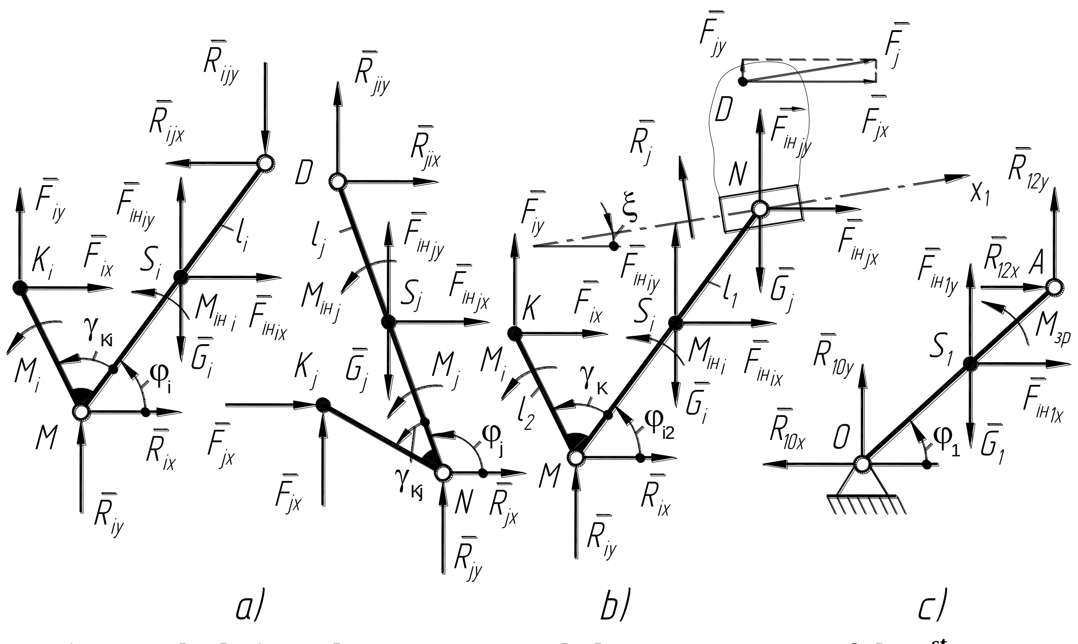


Fig. 1. Calculation schemes: a) Second-class Assur group of the 1st type; b) Second-class Assur group of the 2nd type; c) Mechanism of the I class

To define the components of the reaction force R_i we projected all the forces related to link i to the coordinate axes:

$$R_{ix} = R_{jix} - F_{ix} - F_{\text{ін}ix}; R_{iy} = R_{jiy} - F_{iy} - F_{\text{ін}iy} + G_i; R_i = \sqrt{R_{ix}^2 + R_{iy}^2}. \qquad (4)$$

Components of the reaction force R_j in other external kinematic pair N:

$$R_{jx} = -R_{jix} - F_{jx} - F_{\text{ін}jx};\ R_{jy} = -R_{jiy} - F_{jy} - F_{\text{ін}jy} + G_2;\ R_j = \sqrt{R_{jx}^2 + R_{jy}^2}. \quad (5)$$

In the unified analysis program of the group we take into account that:

$$l_j \cos\varphi_j = x_D - x_N; l_j \sin\varphi_j = y_D - y_N; l_i \cos\varphi_i = x_D - x_M; l_i \sin\varphi_i = y_D - y_M. (6)$$

Kinetostatic analysis of the Assur group of the 1st type in Mathcad.

The initial data for the calculation is grouped into matrices M_1, A, M_2, F.

Besides, we must specify the values of MD (see Fig. 1, a) and ω_1 – angular velocity of crank OA rotation.

$$M_1 = \begin{pmatrix} x_M & x_N & MS_i & MK_i \\ y_M & y_N & \gamma_{Si} & \gamma_{Ki} \\ x''_M & x''_N & NS_j & NK_j \\ y''_M & y''_N & \gamma_{Sj} & \gamma_{Kj} \end{pmatrix} \quad A = \begin{pmatrix} \varphi_i & \varphi_j \\ \varphi'_i & \varphi'_j \\ \varphi''_i & \varphi''_j \end{pmatrix} \quad M_2 = \begin{pmatrix} m_i \\ m_j \\ J_{Si} \\ J_{Sj} \end{pmatrix} \quad F = \begin{pmatrix} M_i & M_j \\ F_{ix} & F_{jx} \\ F_{iy} & F_{jy} \end{pmatrix}$$

The matrix M_1 includes such parameters as x_M, y_M, x_N, y_N – the coordinates of corresponding points M and N on the scheme; $x''_M, y''_M, x''_N, y''_N$ – the analogs of acceleration (defined preliminarily during the kinematic analysis [4,5]); $MS_i, NS_j, \gamma_{Si}, \gamma_{Sj}$ – positions of the mass centers of the links of structural group (see Fig. 1, a); $MK_i, \gamma_{Ki}, NK_j, \gamma_{Kj}$ – parameters which define positions of the points K_i and K_j of external loads application.

The matrix A includes angles φ_i, φ_j which define the positions of the links of the structural group (calculated during the kinematic analysis [4,5]) and analogs of their velocity and acceleration. The matrix M_2 includes masses m_i, m_j of the links and their moments of inertia J_{Si}, J_{Sj}. The matrix F defines external loads on the links of the group: moments M_i, M_j and forces (projected on coordinate axes). The entire algorithm of kinetostatic analysis of the II class Assur group of the 1st type is made up as the following Mathcad-subroutine:

$$FG1(M_1, A, MD, M_2, F, \omega_1) :=$$

$$\begin{array}{|l}
\begin{pmatrix} x_M & x_N & MS_i & MK_i & y_M & y_N & \gamma_{Si} \\ x''_M & x''_N & NS_j & NK_j & y''_M & y''_N & \gamma_{Sj} \end{pmatrix} \leftarrow \begin{pmatrix} M1_{1,1} & M1_{1,2} & M1_{1,3} & M1_{1,4} & M1_{2,1} & M1_{2,2} & M1_{2,3} \\ M1_{3,1} & M1_{3,2} & M1_{3,3} & M1_{3,4} & M1_{4,1} & M1_{4,2} & M1_{4,3} \end{pmatrix} \\
\begin{pmatrix} \varphi_i & \varphi'_i & \varphi''_i & \varphi_j & \varphi'_j & \varphi''_j & m_i & m_j & \gamma_{Ki} \end{pmatrix} \leftarrow \begin{pmatrix} A_{1,1} & A_{2,1} & A_{3,1} & A_{1,2} & A_{2,2} & A_{3,2} & M2_{1,1} & M2_{2,1} & M1_{2,4} \end{pmatrix} \\
\begin{pmatrix} M_i & F_{ix} & F_{iy} & M_j & F_{jx} & F_{jy} & J_{Si} & J_{Sj} & \gamma_{Kj} \end{pmatrix} \leftarrow \begin{pmatrix} F_{1,1} & F_{2,1} & F_{3,1} & F_{1,2} & F_{2,2} & F_{3,2} & M2_{3,1} & M2_{4,1} & M1_{4,4} \end{pmatrix} \\
\begin{pmatrix} S_{iM} & S_{jM} \end{pmatrix} \leftarrow \left[KDTw\left[\begin{pmatrix} x_M & x''_M \\ y_M & y''_M \end{pmatrix}, MS_i, \gamma_{Si}, \begin{pmatrix} \varphi_i & \varphi'_i & \varphi''_i \end{pmatrix}\right] \; KDTw\left[\begin{pmatrix} x_N & x''_N \\ y_N & y''_N \end{pmatrix}, NS_j, \gamma_{Sj}, \begin{pmatrix} \varphi_j & \varphi'_j & \varphi''_j \end{pmatrix}\right] \right] \\
\begin{pmatrix} S_i & S''_i & S_j & S''_j & M_{in_i} & M_{in_j} \end{pmatrix} \leftarrow \begin{pmatrix} S_{iM_{1,1}} & S_{iM_{1,2}} & S_{jM_{1,1}} & S_{jM_{1,2}} & -J_{Si} \cdot \varphi''_i \cdot \omega_1^2 & -J_{Sj} \cdot \varphi''_j \cdot \omega_1^2 \end{pmatrix} \\
\begin{pmatrix} D & K_i & K_j \end{pmatrix} \leftarrow \begin{pmatrix} KDTs(x_M, y_M, MD, 0, \varphi_i) & KDTs(x_M, y_M, MK_i, \gamma_{Ki}, \varphi_i) & KDTs(x_N, y_N, NK_j, \gamma_{Kj}, \varphi_j) \end{pmatrix} \\
\begin{pmatrix} F_{in_i_x} & F_{in_i_y} & F_{in_j_x} & F_{in_j_y} \end{pmatrix} \leftarrow \begin{pmatrix} -m_i \cdot S''_{i_x} \cdot \omega_1^2 & -m_i \cdot S''_{i_y} \cdot \omega_1^2 & -m_j \cdot S''_{j_x} \cdot \omega_1^2 & -m_j \cdot S''_{j_y} \cdot \omega_1^2 \end{pmatrix} \\
\begin{pmatrix} \Delta_{XDM} & \Delta_{YDM} & \Delta_{XDN} & \Delta_{YDN} \end{pmatrix} \leftarrow \begin{pmatrix} D_1 - x_M & D_2 - y_M & D_1 - x_N & D_2 - y_N \end{pmatrix} \\
B_i \leftarrow F_{in_i_x} \cdot (S_{i_y} - y_M) - M_{in_i} - (F_{in_i_y} - 9.81 \cdot m_i) \cdot (S_{i_x} - x_M) + F_{ix} \cdot (K_{i_y} - y_M) - M_i - F_{iy} \cdot (K_{i_x} - x_M) \\
B_j \leftarrow F_{in_j_x} \cdot (S_{j_y} - y_N) - M_{in_j} - (F_{in_j_y} - 9.81 \cdot m_j) \cdot (S_{j_x} - x_N) + F_{jx} \cdot (K_{j_y} - y_N) - M_j - F_{jy} \cdot (K_{j_x} - x_N) \\
\begin{pmatrix} R_{ji_x} & R_{ji_y} \end{pmatrix} \leftarrow \begin{pmatrix} \dfrac{B_i \cdot \Delta_{XDN} + B_j \cdot \Delta_{XDM}}{\Delta_{YDM} \cdot \Delta_{XDN} - \Delta_{YDN} \cdot \Delta_{XDM}} & \dfrac{B_j \cdot \Delta_{YDM} + B_i \cdot \Delta_{YDN}}{\Delta_{YDM} \cdot \Delta_{XDN} - \Delta_{YDN} \cdot \Delta_{XDM}} \end{pmatrix} \\
\begin{pmatrix} R_{i_x} & R_{i_y} \end{pmatrix} \leftarrow \left[-F_{in_i_x} - F_{ix} + R_{ji_x} \quad -F_{in_i_y} - F_{iy} + (R_{ji_y} + 9.81 \cdot m_i) \right] \\
\begin{pmatrix} R_{j_x} & R_{j_y} \end{pmatrix} \leftarrow \begin{pmatrix} -F_{in_j_x} - F_{jx} - R_{ji_x} & -F_{in_j_y} - F_{jy} - R_{ji_y} + 9.81 \cdot m_j \end{pmatrix} \\
\begin{pmatrix} R_{ji} & R_i & R_j & F_{in_i} & F_{in_j} & M_{in_i} & M_{in_j} \end{pmatrix}
\end{array}$$

As a result, the reactions forces R_{ji}, R_i, R_j in kinematic pairs, inertia forces $F_{\text{ін}i}, F_{\text{ін}j}$, moments of inertia $M_{\text{ін}i}, M_{\text{ін}j}$ of the links are defined. In the program which is shown above, such auxiliary functions are used:

1. Function for displacement determination of any additional point of link:

$$KDTs(F_x, F_y, MS_i, \gamma_{Si}, \varphi) := \begin{pmatrix} F_x + MS_i \cdot \cos(\varphi + \gamma_{Si}) & F_y + MS_i \cdot \sin(\varphi + \gamma_{Si}) \end{pmatrix}$$

2. Determination of the angle between vectors:

$$\text{FindAngle}(Fx,Fy) := \begin{cases} 0 & \text{if } Fx = 0 \wedge Fy = 0 \\ \text{atan2}(Fx,Fy) & \text{otherwise} \end{cases}$$

3. Displacement and analogs of acceleration determination for any additional point of a link:

$$M1 = (x_M \;\; y_M \;\; x''_M \;\; y''_M) \qquad M2 = (\varphi_T \;\; \varphi'_T \;\; \varphi''_T)$$

$$\begin{aligned}
\text{KDTw}(M_1, MS_i, \gamma_{Si}, M_2) := \; & (x_M \;\; x''_M \;\; y_M \;\; y''_M) \leftarrow (M1_{1,1} \;\; M1_{1,2} \;\; M1_{1,3} \;\; M1_{1,4}) \\
& (\varphi_T \;\; \varphi'_T \;\; \varphi''_T) \leftarrow (M2_{1,1} \;\; M2_{1,2} \;\; M2_{1,3}) \\
& (S_{i_x} \;\; S_{i_y}) \leftarrow (x_M + MS_i \cdot \cos(\varphi_T + \gamma_{Si}) \;\; y_M + MS_i \cdot \sin(\varphi_T + \gamma_{Si})) \\
& S''_{i_x} \leftarrow x''_M - MS_i \cdot \varphi''_T \cdot \sin(\varphi_T + \gamma_{Si}) - MS_i \cdot (\varphi'_T)^2 \cdot \cos(\varphi_T + \gamma_{Si}) \\
& S''_{i_y} \leftarrow y''_M + MS_i \cdot \varphi''_T \cdot \cos(\varphi_T + \gamma_{Si}) - MS_i \cdot (\varphi'_T)^2 \cdot \sin(\varphi_T + \gamma_{Si}) \\
& (S_i \;\; S''_i)
\end{aligned}$$

4. Function for the rotation of the coordinate system on specified angle:

$$\begin{aligned}
\text{PKTs}(x_{M2}, y_{M2}, x''_{M2}, y''_{M2}, \xi) := \; & (M_x \;\; M_y) \leftarrow (x_{M2} \cdot \cos(\xi) - y_{M2} \cdot \sin(\xi) \;\; y_{M2} \cdot \cos(\xi) + x_{M2} \cdot \sin(\xi)) \\
& (M''_x \;\; M''_y) \leftarrow (x''_{M2} \cdot \cos(\xi) - y''_{M2} \cdot \sin(\xi) \;\; y''_{M2} \cdot \cos(\xi) + x''_{M2} \cdot \sin(\xi)) \\
& (M \;\; M'')
\end{aligned}$$

Assur group of the 2nd type. Structural group of the 2nd type is shown on the Fig. 1, b which includes the external loads $\bar{F}_i, \bar{F}_j$ which are applied to the links and are shown as projections to the coordinate axes; the moments M_i, M_j and the gravity forces $\bar{G}_i, \bar{G}_j$. Inertia forces of the links $\bar{F}_{\text{ін}i}, \bar{F}_{\text{ін}j}$ and the moment of inertia forces $M_{\text{ін}i}$ relative to the center of mass are calculated according to the known kinematic characteristics of the structural group. Thus, we receive:

$$R_j = \left[\left(F_{\text{ін}jx} + F_{jx}\right) l_1 \sin\varphi_{i2} - \left(F_{\text{ін}jy} + F_{jy} + G_j\right) l_1 \cos\varphi_{i2} + b\right] \Big/ l_1 \cos\left(\varphi_{i2} - \xi\right), \quad (7)$$

where

$$b = F_{ix} l_2 \sin\left(\varphi_{i2} + \gamma_K\right) - F_{iy} l_2 \cos\left(\varphi_{i2} + \gamma_K\right) - M_i - M_{\text{ін}i} - \left(F_{\text{ін}iy} - G_i\right) l_{MS_i} \times$$
$$\times \cos\varphi_{i2} + F_{\text{ін}ix} l_{MS_i} \sin\varphi_{i2};$$

$$R_{jix} = R_j \sin\xi - F_{jx} - F_{\text{ін}\,jx};\; R_{jiy} = -R_j \cos\xi - F_{jy} - F_{\text{ін}\,jy} + G_j. \qquad (8)$$

For the coupler MN (see Fig. 1, b) we can make two equations of the forces projections sum on coordinate axes and define the reaction $\bar{R}_i$:

$$R_{ix} = R_{jix} - F_{ix} - F_{\text{ін}ix};\; R_{iy} = R_{jiy} - F_{iy} - F_{\text{ін}iy} + G_i;\; R_i = \sqrt{R_{ix}^2 + R_{iy}^2}. \qquad (9)$$

Kinetostatic analysis of the Assur group of the 2nd type in Mathcad.

The initial data is grouped into the matrices M_1, M_2, F in the same way as it was shown above for the kinetostatic analysis of the group of the 1st type. Besides, in the initial data we must also specify the values of MN (see Fig.1, b) and ω_1 – angular velocity of crank rotation. The matrices of the initial data are:

$$M_1 = \begin{pmatrix} x_M & y_M & x''_M & y''_M \end{pmatrix} \quad M_2 = \begin{pmatrix} \varphi_{i2} & MK & m_i \\ \varphi'_i & \gamma_K & m_j \\ \varphi''_i & MS_i & J_{Si} \end{pmatrix} \quad F = \begin{pmatrix} M_i & \xi \\ F_{ix} & F_{jx} \\ F_{iy} & F_{jy} \end{pmatrix}$$

The main subroutine of the group kinetostatic analysis is shown below.

$$FG2\left(M_1, M_2, MN, F, \omega_1\right) :=$$

$$\left(x_M\ y_M\ x''_M\ y''_M\ \varphi_{i2}\ MK\ m_i\right) \leftarrow \left(M1_{1,1}\ M1_{1,2}\ M1_{1,3}\ M1_{1,4}\ M2_{1,1}\ M2_{1,2}\ M2_{1,3}\right)$$

$$\left(\varphi'_i\ \gamma_K\ m_j\ \varphi''_i\ MS_i\ J_{Si}\ \xi\right) \leftarrow \left(M2_{2,1}\ M2_{2,2}\ M2_{2,3}\ M2_{3,1}\ M2_{3,2}\ M2_{3,3}\ F_{1,2}\right)$$

$$\left(M_i\ F_{ix}\ F_{iy}\ F_{jx}\ F_{jy}\ \varphi_i\ K\right) \leftarrow \left(F_{1,1}\ F_{2,1}\ F_{3,1}\ F_{2,2}\ F_{3,2}\ \varphi_{i2}+\xi\ KDTs\left(x_M, y_M, MK, \gamma_K, \varphi_{i2}+\xi\right)\right)$$

$$\left(S_{iM}\ N_M\right) \leftarrow \left[KDTw\left[\begin{pmatrix} x_M & x''_M \\ y_M & y''_M \end{pmatrix}, MS_i, 0, \left(\varphi_i\ \varphi'_i\ \varphi''_i\right)\right]\ KDTw\left[\begin{pmatrix} x_M & x''_M \\ y_M & y''_M \end{pmatrix}, MN, 0, \left(\varphi_i\ \varphi'_i\ \varphi''_i\right)\right]\right]$$

$$\left(S_i\ S''_i\ N\ N''\ M_{\text{ін}\_i}\right) \leftarrow \left(S_{iM_{1,1}}\ S_{iM_{1,2}}\ N_{M_{1,1}}\ N_{M_{1,2}}\ -J_{Si}\cdot\varphi''_i\cdot\omega_1^{2}\right)$$

$$\left(F_{\text{ін}\_i_x}\ F_{\text{ін}\_i_y}\ F_{\text{ін}\_j_x}\ F_{\text{ін}\_j_y}\right) \leftarrow \left(-m_i\cdot S''_{i_x}\cdot\omega_1^{2}\ \ -m_i\cdot S''_{i_y}\cdot\omega_1^{2}\ \ -m_j\cdot N''_1\cdot\omega_1^{2}\ \ -m_j\cdot N''_2\cdot\omega_1^{2}\right)$$

$$B_i \leftarrow -F_{iy}\cdot\left(K_x - x_M\right) - M_i - M_{\text{ін}\_i} - \left(F_{\text{ін}\_i_y} - 9.81\cdot m_i\right)\cdot\left(S_{i_x} - x_M\right) + \left[F_{\text{ін}\_i_x}\cdot\left(S_{i_y} - y_M\right) + F_{ix}\cdot\left(K_y - y_M\right)\right]$$

$$\left(\Delta_{YNM}\ \Delta_{XNM}\right) \leftarrow \left(N_y - y_M\ \ N_x - x_M\right)$$

$$R_j \leftarrow \frac{B_i + \left(F_{\text{ін}\_j_x} + F_{jx}\right)\cdot\Delta_{YNM} - \left(F_{\text{ін}\_j_y} + F_{jy} - 9.81\cdot m_j\right)\cdot\Delta_{XNM}}{\Delta_{YNM}\cdot\sin(\xi) + \Delta_{XNM}\cdot\cos(\xi)}$$

$$\left(R_{ji_x}\ R_{ji_y}\right) \leftarrow \left[R_j\cdot\sin(\xi) - F_{\text{ін}\_j_x} - F_{jx}\ \ -R_j\cdot\cos(\xi) - \left(F_{\text{ін}\_j_y} - F_{jy}\right) + 9.81\cdot m_j\right]$$

$$\left(R_{i_x}\ R_{i_y}\right) \leftarrow \left(R_{ji_x} - F_{\text{ін}\_i_x} - F_{ix}\ \ R_{ji_y} - F_{\text{ін}\_i_y} - F_{iy} + 9.81\cdot m_i\right)$$

$$\left(R_i\ R_{ji}\ F_{\text{ін}\_i}\ F_{\text{ін}\_j}\ R_j\ M_{\text{ін}\_i}\right)$$

Mechanism of the I class. Let's consider the case where the input link of mechanism forms the rotational kinematic pair (see Fig. 1, c). The equations for the equilibrium moment $M_{зр}$ and reaction force $\bar{R}_{10}$:

$$M_{зр} + M_A\left(R_{12x}\right) + M_A\left(R_{12y}\right) + M_A\left(F_{\text{ін}1x}\right) + M_A\left(F_{\text{ін}1y}\right) + M_A\left(G_1\right) + M_{\text{ін}1} = 0; \quad (10)$$

$$M_A\left(R_{12x}\right) = -R_{12x} l_{OA} \sin\varphi_1; M_A\left(R_{12y}\right) = R_{12y} l_{OA} \cos\varphi_1; M_A\left(F_{\text{ін}1x}\right) = F_{\text{ін}1x}\, l_{OS_1} \times$$
$$\times \sin\varphi_1; M_A\left(F_{\text{ін}1y}\right) = F_{\text{ін}1y}\, l_{OS_1} \cos\varphi_1;\ M_A\left(G_1\right) = -G_1 l_{OS_1} \cos\varphi_1;\ M_{\text{ін}1} = -J_{S_1}\varepsilon_1. \quad (11)$$

$$R_{10} = \sqrt{R_{10x}^2 + R_{10y}^2}; R_{10x} = R_{12x} - F_{ін1x};\ \ R_{10y} = -R_{12x} - F_{ін1y} + G_1; \quad (12)$$

Kinetostatic analysis of the mechanism of the I class in Mathcad.

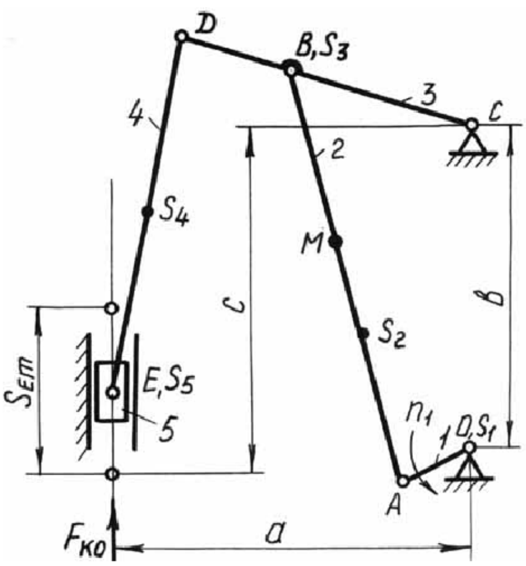
The parameters of the mechanism are shown on Fig.1, c. The Mathcad-subroutine for the kinetostatic analysis of this mechanism is shown below:

$$FM1\left(R_{21x}, R_{21y}, M, x_0, y_0, OA, \varphi_1, OS_1, m_1, \omega_1\right) :=$$

$$\left|\begin{array}{l}
\left(x_A\ \ y_A\ \ S_{i_x}\ \ S_{i_y}\right) \leftarrow \left(OA\cdot\cos(\varphi_1)\ \ OA\cdot\sin(\varphi_1)\ \ OS_1\cdot\cos(\varphi_1)\ \ OS_1\cdot\sin(\varphi_1)\right) \\
\left(F_{\text{iн_1}_x}\ \ F_{\text{iн_1}_y}\right) \leftarrow \left(m_1\cdot OS_1\cdot\omega_1^{\,2}\cdot\cos(\varphi_1)\ \ m_1\cdot OS_1\cdot\omega_1^{\,2}\cdot\sin(\varphi_1)\right) \\
F \leftarrow \begin{bmatrix} -R_{21x}\cdot(y_A - y_0) + R_{21y}\cdot(x_A - x_0) - F_{\text{iн_1}_y}\cdot\left(S_{i_x} - x_0\right) \ldots \\ +F_{\text{iн_1}_x}\cdot\left(S_{i_y} - y_0\right) + 9.81\cdot m_1\cdot\left(S_{i_x} - x_0\right) - M \end{bmatrix}\cdot OA^{-1} \\
\left(R_{10_x}\ \ R_{10_y}\right) \leftarrow \left(F\cdot\sin(\varphi_1) - F_{\text{iн_1}_x} + R_{21x}\ \ 9.81\cdot m_1 - F_{\text{iн_1}_y} + R_{21y} - F\cdot\cos(\varphi_1)\right) \\
\left(F_{\text{iн_1}}\ \ R_{10}\ \ F\right)
\end{array}\right.$$

Let’s consider an example of using the shown unified algorithms to perform the kinetostatic analysis of six-link linkage mechanism of press. Mathcad-program is shown below. Notice that kinematic characteristics of the mechanism are considered to be predefined. The algorithms of the kinematic analysis of the mechanisms of II and III class were considered in [4-6].

Kinetostatic analysis of mechanism of press



Dimensions of the links

$a := 0.8 \quad b := 1.3 \quad OA := 0.15 \quad AB := 1.35$

$BC := 0.5 \quad CD := 0.66 \quad DE := 0.57$

Positions of the mass centers

$OS_1 := 0.25\cdot OA \quad AS_2 := 0.5\cdot AB \quad DS_4 := 0.4\cdot DE$

Mass of the links

$m_1 := \frac{360}{9.81} \quad m_2 := \frac{570}{9.81} \quad m_3 := \frac{600}{9.81}$

$m_4 := m_2 \quad m_5 := \frac{425}{9.81}$

Moments of inertia of the links

$J_{S1} := 0.1 \quad J_{S2} := 0.16 \quad J_{S3} := 0.2 \quad J_{S4} := J_{S2}$

Start position of crank $\varphi_0 := 108.85\cdot\frac{\pi}{180}$

Quantity of investigated positions $N := 180$

Crank rotating frequency, RPM $n_1 := 100 \quad \omega_1 := \frac{\pi\cdot n_1}{30}$ Crank angle $\varphi_1 := \varphi_0, \varphi_0 + 2\cdot\frac{\pi}{N} .. \varphi_0 + 2\cdot\pi$

Kinetostatic analysis of Assur group of II class and 2nd type (4-5)

$$FG2\_Results(\varphi_1) := FG2\left[\begin{pmatrix} x_D(\varphi_1) \\ y_D(\varphi_1) \\ \frac{d^2}{d\varphi_1^2}x_D(\varphi_1) \\ \frac{d^2}{d\varphi_1^2}y_D(\varphi_1) \end{pmatrix}, \begin{pmatrix} \varphi_{i2}(\varphi_1) & 0 & m_4 \\ \frac{d}{d\varphi_1}\varphi_{i2}(\varphi_1) & 0 & m_5 \\ \frac{d^2}{d\varphi_1^2}\varphi_{i2}(\varphi_1) & DS_4 & J_{S4} \end{pmatrix}, DE, \begin{pmatrix} 0 & \frac{3\cdot\pi}{2} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \omega_1\right]$$

$$R(\varphi_1, i) := \begin{array}{|l} \text{if } i \le 4 \\ \quad \begin{array}{|l} (x\ \ y) \leftarrow \left[(FG2\_Results(\varphi_1)_{1,i})_x \ \ (FG2\_Results(\varphi_1)_{1,i})_y\right] \\ (Result_F\ \ Result_\alpha) \leftarrow \left(\sqrt{x^2+y^2}\ \ FindAngle(x,y)\right) \end{array} \\ Result \leftarrow (FG2\_Results(\varphi_1)_{1,i}) \ \ \text{otherwise} \\ Result \end{array}$$

$$R_{43}(\varphi_1) := R(\varphi_1,1)_F$$
$$R_{54}(\varphi_1) := R(\varphi_1,2)_F$$
$$F_{iH4}(\varphi_1) := R(\varphi_1,3)_F$$
$$F_{iH5}(\varphi_1) := R(\varphi_1,4)_F$$
$$R_{50}(\varphi_1) := R(\varphi_1,5)$$
$$M_{iH4}(\varphi_1) := R(\varphi_1,6)$$

$$\alpha_{54}(\varphi_1) := R(\varphi_1,2)_\alpha \quad \alpha_{43}(\varphi_1) := R(\varphi_1,1)_\alpha \quad \alpha_{FiH4}(\varphi_1) := R(\varphi_1,3)_\alpha \quad \alpha_{FiH5}(\varphi_1) := R(\varphi_1,4)_\alpha$$

$$R_{34x}(\varphi_1) := -R_{43}(\varphi_1)\cdot\cos(\alpha_{43}(\varphi_1)) \qquad R_{34y}(\varphi_1) := -R_{43}(\varphi_1)\cdot\sin(\alpha_{43}(\varphi_1))$$

Kinetostatic analysis of Assur group of II class and 1st type (2-3)

$$FG1\_Results(\varphi_1) :=$$

$$FG1\left[\begin{pmatrix} x_A(\varphi_1) & 0 & AS_2 & 0 \\ y_A(\varphi_1) & b & 0 & 0 \\ \frac{d^2}{d\varphi_1^2}x_A(\varphi_1) & 0 & BC & CD \\ \frac{d^2}{d\varphi_1^2}y_A(\varphi_1) & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \varphi_2(\varphi_1) & \varphi_3(\varphi_1) \\ \frac{d}{d\varphi_1}\varphi_2(\varphi_1) & \frac{d}{d\varphi_1}\varphi_3(\varphi_1) \\ \frac{d^2}{d\varphi_1^2}\varphi_2(\varphi_1) & \frac{d^2}{d\varphi_1^2}\varphi_3(\varphi_1) \end{pmatrix}, AB, \begin{pmatrix} m_2 \\ m_3 \\ J_{S2} \\ J_{S3} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & R_{34x}(\varphi_1) \\ 0 & R_{34y}(\varphi_1) \end{pmatrix}, \omega_1\right]$$

$$R(\varphi_1,i) := \begin{array}{|l} \text{if } i \le 5 \\ \quad \begin{array}{|l} (x\ \ y) \leftarrow \left[(FG1\_Results(\varphi_1)_{1,i})_x \ \ (FG1\_Results(\varphi_1)_{1,i})_y \right] \\ (Result_F\ \ Result_\alpha) \leftarrow \left(\sqrt{x^2+y^2}\ \ FindAngle(x,y) \right) \end{array} \\ Result \leftarrow (FG1\_Results(\varphi_1)_{1,i}) \ \ \text{otherwise} \\ Result \end{array}$$

$$R_{32}(\varphi_1) := R(\varphi_1,1)_F \quad R_{30}(\varphi_1) := R(\varphi_1,3)_F \quad F_{iн3}(\varphi_1) := R(\varphi_1,5)_F \quad R_{21}(\varphi_1) := R(\varphi_1,2)_F$$

$$F_{iн2}(\varphi_1) := R(\varphi_1,4)_F \quad M_{iн2}(\varphi_1) := R(\varphi_1,6) \quad M_{iн3}(\varphi_1) := R(\varphi_1,7)$$

$$\alpha_{32}(\varphi_1) := R(\varphi_1,1)_\alpha \quad \alpha_{30}(\varphi_1) := R(\varphi_1,3)_\alpha \quad \alpha_{Fiн3}(\varphi_1) := R(\varphi_1,5)_\alpha$$

$$\alpha_{21}(\varphi_1) := R(\varphi_1,2)_\alpha \quad \alpha_{Fiн2}(\varphi_1) := R(\varphi_1,4)_\alpha$$

$$R_{21x}(\varphi_1) := R_{21}(\varphi_1)\cdot\cos(\alpha_{21}(\varphi_1)) \qquad R_{21y}(\varphi_1) := R_{21}(\varphi_1)\cdot\sin(\alpha_{21}(\varphi_1))$$

Kinetostatic analysis of the mechanism of the I class (0-1)

$$FM1\_Results(\varphi_1) := FM1(R_{21x}(\varphi_1), R_{21y}(\varphi_1), 0, 0, 0, OA, \varphi_1, OS_1, m_1, \omega_1)$$

$$R(\varphi_1,i) := \begin{array}{|l} \text{if } i \le 2 \\ \quad \begin{array}{|l} (x\ \ y) \leftarrow \left[(FM1\_Results(\varphi_1)_{1,i})_x \ \ (FM1\_Results(\varphi_1)_{1,i})_y \right] \\ (Result_F\ \ Result_\alpha) \leftarrow \left(\sqrt{x^2+y^2}\ \ FindAngle(x,y) \right) \end{array} \\ Result \leftarrow (FM1\_Results(\varphi_1)_{1,i}) \ \ \text{otherwise} \\ Result \end{array}$$

$$F_{iн1}(\varphi_1) := R(\varphi_1,1)_F \quad \alpha_{Fiн1}(\varphi_1) := R(\varphi_1,1)_\alpha \quad R_{10}(\varphi_1) := R(\varphi_1,2)_F \quad \alpha_{R10}(\varphi_1) := R(\varphi_1,2)_\alpha$$

Equilibrium force $F_{3p}(\varphi_1) := R(\varphi_1,3)$

The results of calculations (reaction and inertia forces, moments of inertia forces, etc) are defined in Mathcad as functions (with the angle of crank rotation φ_1 as parameter). Samples of the kinetostatic analy8sis results are shown below.

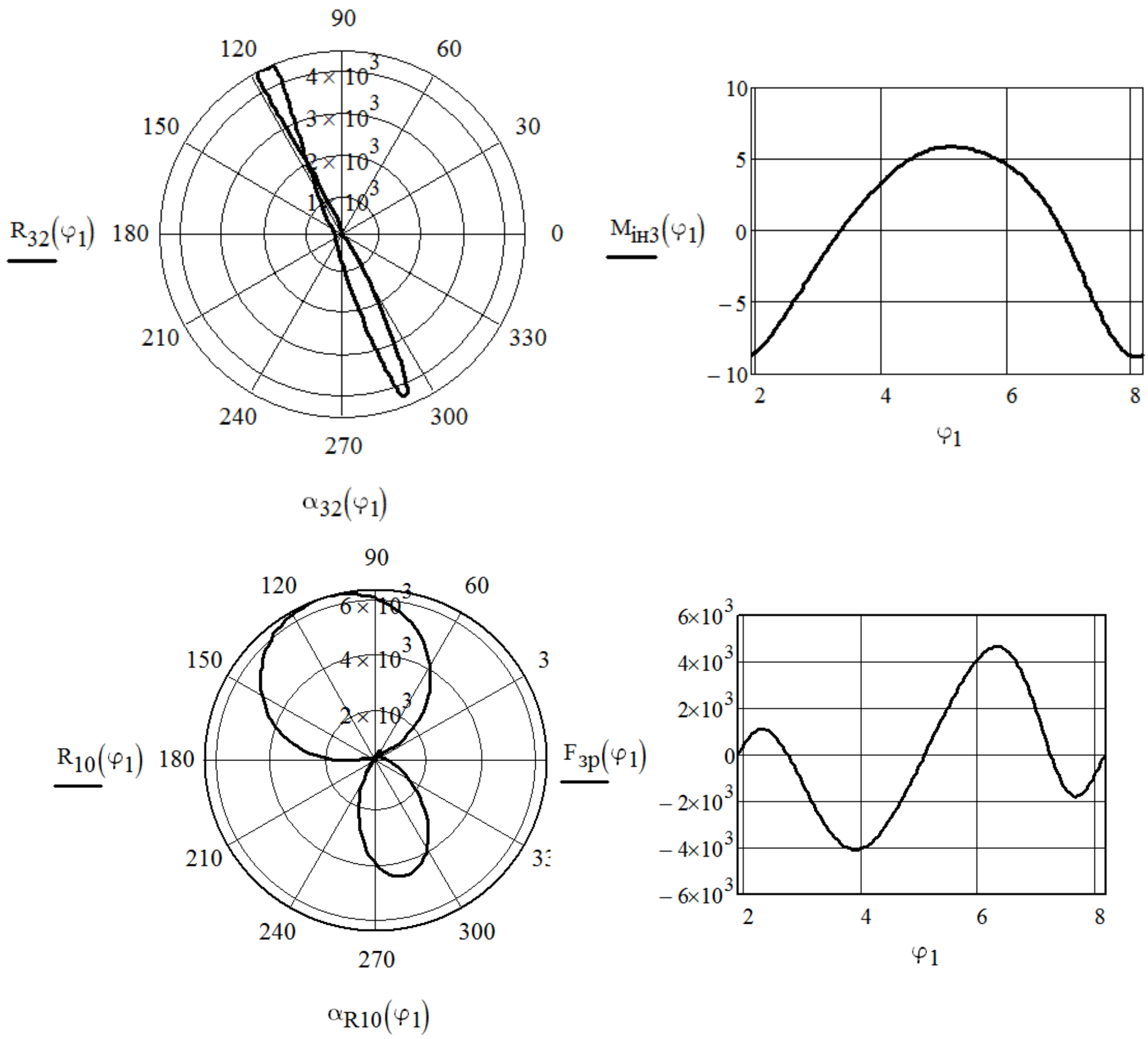


Fig. 2.The samples of calculated results of kinetostatic analysis

Conclusions

The main advantage of the unified algorithms usage is that they can be applied not only for particular linkage mechanisms, but for the different mechanisms which include all the possible 5 types of II class Assur groups with any numbers of links. Besides, it enables to use both symbolic and numerical power of Mathcad to make an optimization procedure and results visualization.

One of the most difficult tasks in the theory of mechanisms and machines is the synthesis of linkage mechanisms. In comparison with 3D Mechanical CADs, Mathcad is more flexible in optimization and can be easily used for implementation of the synthesis theories and engineering methods. Suggested unified algorithms of kinetostatic analysis can be used to verify different combinations of input parameters of mechanism which helps find better solution.

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